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MONTHLY NOTICES

OF THE

ROYAL ASTRONOMICAL SOCIETY.

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W. H. MAW, Esq., PRESIDENT, in the Chair.

Ralph Falcon, M.A. Oxon., Camerton Hall, Workington, Cumberland; and
Arthur Grant Stillhamer, Yerkes Observatory, Williams Bay, Wisconsin, U.S.A.,

were balloted for and duly elected Fellows of the Society.

The following candidates were proposed for election as Fellows of the Society, the names of the proposers from personal knowledge being appended:—

Rodney Boyce, Soudan Survey Department, c/o the Royal Colonial Institute, Northumberland Avenue, S.W. (proposed by E. P. Cotton);
Arthur Cleminson, Deputy Commissioner of Lands, Lagos, West Africa (proposed by E. P. Cotton);
George Innes, M.P.S., Chemist and Optician, Olive Bank, Edinburgh (proposed by Rev. John Spence);
Arthur Kent Lucke, Suez Canal Company's Service, Transit Department, Ismaïlia, Egypt (proposed by E. W. Maunder); and
George Street, M.A., Tutor, Merton House, Southwick, Sussex (proposed by Arthur R. Hinks).

One hundred and sixty-five presents were announced as having been received since the last meeting, including, amongst others:—

Galileo, Opere, Edizio Nazionale, vols. 16, 17, 18, presented by the Italian Government; Catalog der Astronomischen Gesellschaft, Abtheilung ii. Stück 1, $-2°$ bis $-6°$ (Strassburg), presented by Professor Becker [and another copy presented by the *Astronomische Gesellschaft*]; H. Dennis Taylor, A System of Applied Optics, presented by the author.

Seventy-eight charts of the Astrographic Chart of the Heavens, presented by the Royal Observatory, Greenwich; and ninety-six charts, presented by the Paris Observatory.

Two enlarged transparencies and lantern slide from negatives of the total eclipse of August 1905, taken at Alcalá Chisvert, Spain, presented by Count de la Baume Pluvinel.

The Early Eclipses of the Sun and Moon. By E. Nevill.

With the view of furnishing Mr Cowell with the further information he desires (*Monthly Notices*, vol. lxvi. p. 474), I have extracted from my memoir on the Observed Errors of Hansen's *Tables de la lune* my investigation of the results yielded by the observations of the ancient Eclipses of the Moon contained in Ptolemy's *Almagest*.

The eclipses and errors of Hansen's Tables are as follow:—

	Date.	Place.	Phase.	Error of Hansen's Tables.	Weight.	Remarks.
1	−720 March 19	Babylon	Beginning	-23^m	2	Total
2	−719 March 8	,,	Middle	−63	½	Mag. = 0·25 on S
3	−719 Sept. 1	,,	Beginning	−43	2	Mag. = 0·55 on N
4	−620 April 21	,,	Beginning	−44	3	Mag. = 0·25 on S
5	−522 July 16	,,	Beginning	−93	1	Mag. = 0·50 on N
6	−501 Nov. 19	,,	Beginning	−28	1	Mag. = 0·25 on S
7	−490 April 25	,,	Beginning	−28	1	Mag. = 0·17 on S
8	−382 Dec. 22	Athens	Beginning	−51	½	Small. Place uncertain
9	−381 June 18	Babylon	Beginning	−43	2	Partial
9a	,,	,,	End	−77	1	Duration $3^h\ 0^m$
10	−381 Dec. 12	,,	Beginning	−59	1	Total
11	−200 Sept. 22	,,	Beginning	−26	1	Partial
11a	,,	,,	End	−30	2	Duration $3^h\ 2^m$
12	−199 March 19	Alexandria	Beginning	−38	3	Total
13	−199 Sept. 11	,,	Beginning	−25	2	Total
13a	...	,,	Middle	−19	1	..
14	−173 April 30	,,	Beginning	−44	1	Mag. = 0·58 on N
14a	...	,,	End	−46	1	Duration $2^h\ 43^m$
15	−143 Jan. 27	Rhodes	Beginning	−83	½	Mag. = 0·25 on S
16	+125 May 5	Alexandria	Beginning	−26	2	Mag. = 0·17 on S
17	+133 May 6	,,	Middle	−30	3	Total
18	+134 Oct. 20	,,	Middle	−13	2	Mag. = 0·83 on N
19	+136 March 5	,,	Middle	−48	2	Mag. = 0·50 on N

The data have been taken, primarily, from Professor Newcomb's *Researches on the Motion of the Moon*, but in certain cases I have interpreted the record in a somewhat different manner, on the strength of additional information, and have assumed that the Eclipse No. 8 was observed at Athens, the actual place of observation being left unspecified. This Eclipse No. 8 and Eclipse No. 15 observed at Rhodes stand in a different category to the others, seemingly having been selected from outside Ptolemy's usual authorities for some special reason, possibly because no eclipses observed at either Babylon or Alexandria were observed under the desired conditions. Neither has been used in the discussion which follows.

Writing—

(Tabular—Observed) Longitude $= \Delta L + \sin\alpha \,.\, A_s + \cos\alpha \,.\, A_c + \sin\beta \,.\, B_s + \cos\beta \,.\, B_c + (T - 1800) \,.\, 30'' \,.\, \delta l^* + (T - 1800)^2 \, 1''.0 \, A$

where α denotes the Moon's mean anomaly and β denotes the Moon's mean argument of latitude. Then the preceding data lead to the expressions—

1	$\Delta L = +11'.1$	$+.88\,A$	$+.47\,A_c$	$-.01\,B_s$	$-.99\,B_c$	$+4'.20\,\delta l^*$	$-10''.58\,A$
2	$= +28'.5$	$+.21$	$+.98$	$-.15$	$-.99$	$+4'.20$	$-10''.57$
3	$= +25'.3$	$+.33$	$-.94$	$+.13$	$+.99$	$+4'.20$	$-10''.57$
4	$= +19'.9$	$-.31$	$+.95$	$-.18$	$+.98$	$+4'.03$	$-9''.76$
5	$= +42'.7$	$+.45$	$+.89$	$+.15$	$-.99$	$+3'.87$	$-8''.99$
6	$= +12'.6$	$+.06$	$+.99$	$-.14$	$+.99$	$+3'.84$	$-8''.85$
7	$= +14'.7$	$+.99$	$-.07$	$-.20$	$+.98$	$+3'.82$	$-8''.75$
8	$= +28'.7$	$-.78$	$-.62$	$+.20$	$-.98$	$+3'.65$	$-7''.95$
9	$= +18'.9$	$+.42$	$+.91$	$-.15$	$+.99$	$+3'.64$	$-7''.93$
9a	$= +33'.8$	$+.42$	$+.91$	$-.15$	$+.99$	$+3'.64$	$-7''.93$
10	$= +35'.0$	$-.01$	$-.99$	$+.07$	$-.99$	$+3'.64$	$-7''.93$
11	$= +12'.4$	$-.82$	$+.57$	$-.11$	$+.99$	$+3'.34$	$-6''.66$
11a	$= +14'.3$	$-.82$	$+.57$	$-.11$	$+.99$	$+3'.34$	$-6''.66$
12	$= +20'.3$	$+.97$	$-.22$	$-.02$	$-.99$	$+3'.33$	$-6''.65$
13	$= +13'.5$	$-.96$	$-.29$	$+.00$	$+.99$	$+3'.33$	$-6''.65$
13a	$= +10'.2$	$-.96$	$-.29$	$+.00$	$+.99$	$+3'.33$	$-6''.65$
14	$= +25'.8$	$+.33$	$-.95$	$+.12$	$+.99$	$+3'.29$	$-6''.49$
14a	$= +27'.0$	$+.33$	$-.95$	$+.12$	$+.99$	$+3'.29$	$-6''.49$
15	$= +48'.9$	$+.05$	$-.99$	$-.15$	$-.99$	$+3'.24$	$-6''.27$
16	$= +13'.9$	$-.95$	$-.29$	$-.17$	$+.98$	$+2'.83$	$-4''.68$
17	$= +14'.1$	$-.69$	$+.72$	$+.08$	$-.99$	$+2'.81$	$-4''.63$
18	$= +6'.3$	$+.86$	$+.50$	$+.08$	$+.99$	$+2'.81$	$-4''.63$
19	$= +21'.7$	$+.62$	$-.78$	$+.19$	$-.98$	$+2'.80$	$-4''.62$

Let—

A denote the values obtained from Hansen's Tables, with his

tabular mean motions and secular accelerations replaced by Hansen's finally deduced values for these elements.

B denote the values obtained from Hansen's Tables after replacing his tabular values for the secular accelerations by those derived theoretically from the secular diminution of the eccentricity of the terrestrial orbit, removing his empirical Venus term of long period, and making the necessary corrections to the mean motions requisite to bring the amended Tables as far as possible into accord with the modern observations.

C denote the values obtained from Hansen's Tables after applying the corrections indicated by the discussion of the observations of the Moon made between 1650 and 1890, employing for the secular acceleration of the mean longitude the value $+7''\cdot43$, of the longitude of the node the value $6''\cdot07$, and of the longitude of the lunar perigee the value $-32''\cdot53$.

Then the preceding expressions yield for the tabular errors of the Tables as modified in B and C the values

		Tables B.—Obs.		Tables C.—Obs.	
No.	Date.	ΔL.	ΔE.N.P.D.	ΔL.	ΔE.N.P.D.
		′	′	′	′
1	−720 March 19	−38·1	...	−17·8	...
2	−719 March 8	−19·2	+1·2	+ 3·9	+1·9
3	−719 Sept. 1	−28·9	+1·8	−16·2	+1·1
4	−620 April 21	−24·5	+6·0	− 4·2	+5·6
5	−522 July 16	+ 2·6	−2·4	+21·2	−1·8
6	−501 Nov. 19	−26·3	+4·2	− 7·6	+3·6
7	−490 April 25	−26·2	+5·4	−11·8	+4·8
8	−382 Dec. 22	− 9·5 ?	...	+ 1·5 ?	...
9	−381 June 18	−10·4	+8·3	+ 5·9	+7·8
10	−381 Dec. 12	− 3·9	...	+ 5·8	...
11	−200 Sept. 22	−16·2	+2·0	− 3·3	+1·6
12	−199 March 19	−10·0	...	+ 0·5	...
13	−199 Sept. 11	−18·0	...	− 7·7	...
14	−173 April 30	− 4·1	+2·6	− 4·0	+1·8
15	−140 Jan. 27	+19·4 ?	−1·2	+27·3 ?	−0·8
16	+125 April 5	− 6·0	+2·5	− 0·9	+2·2
17	+135 May 6	− 4·9	...	+ 4·2	...
18	+134 Oct. 20	−12·8	+1·4	− 4·2	+1·7
19	+136 March 5	+ 1·5	−5·0	+ 7·5	−4·7

The Eclipse No. 8, whose place of observation is not stated, but has been assumed to be Athens, yields values fairly accordant with those from the other eclipses. The Eclipse No. 15 observed at Rhodes is quite discordant, indicating either that Zech was correct in supposing the time to be an hour in error, or else that the phase observed was the middle of the eclipse: the first supposition would

render the error in longitude B = − 14′·2 C = − 6′·3, and the second the values B = − 12′·4 C = − 4′·5 ; results in fair accord with those from the other eclipses. Neither eclipse has been included in the investigation of the errors in longitude, however, owing to the inherent uncertainty.

From the preceding, there are derived the following values for the mean outstanding errors of the Tables as amended :—

Epoch.	Tables B.—Obs.	Tables C.—Obs.	Weight.
− 720	ΔL = − 28′·7	= − 15′·2	3
− 620	= − 24·5	= − 4·2	2
− 504	= − 16·6	= + 0·6	2
− 381	= − 8·8	= + 5·9	4
− 192	= − 11·7	= − 1·2	10
+ 125	= − 6·3	= + 0·9	2
+ 134	= − 5·3	= + 2·6	6
Weighted mean	ΔL = − 12·8	= − 0·9	

It is obvious, therefore, that these eclipses cannot be reconciled with the theoretical value of the secular acceleration in mean longitude embodied in Tables B, or with any value less than that adopted in Tables C, or below + 7″·40.

Considering the correction ΔB″ to the secular acceleration of the lunar node, the mean of the results from the observed phases 3, 9a, 11a, and 14 yield the expression—

			Tables B.—Obs.	Tables C.—Obs.
+ 0·70 ΔB″	− 3·62δl*	+ 7·91 A	= − 15′·22	= − 0′·25

whilst the mean of the results from the observed phases 4, 5, 6, 7, 8, 9, 11, 14a, and 16 gives

$$+1{\cdot}38\Delta B'' + 3{\cdot}59\delta l^* - 7{\cdot}78A = +14'{\cdot}20 = +0'{\cdot}03.$$

Hence

$$\Delta B'' - 0{\cdot}03\delta l^* + 0{\cdot}13A = -0''{\cdot}49 = +0''{\cdot}11.$$

The correction obtained is small.

The values of the corrections to the tabular E.N.P.D. regarding them all as of equal weight yield the values—

Epoch.	Obs.	Obs.	Weight.
− 565	Tables B. ΔE.N.P.D. = + 3′·50	Tables C. ΔE.N.P.D. = + 3′·26	7
− 172	= + 0·85	= + 0·65	4
+ 132	= − 0·27	= − 0·37	3

Regarding these as indicating a progressive change in E.N.P.D., they correspond to the values—

Tables B. $\Delta E.N.P.D. = +0'{\cdot}10 - 0'{\cdot}36\,(T-0) + 0'{\cdot}043\,(T-0)^2$

Tables C. $= -0'{\cdot}02 - 0'{\cdot}32\,(T-0) + 0'{\cdot}046\,(T-0)^2$

These values are too large to be real, corresponding to a change of some 20″ per century,—an impossible amount if assumed to extend up to the present time.

If they be regarded as due to the effect of corrections required by the adopted tabular value of the argument of latitude β, necessitated by an erroneous tabular value of the secular acceleration of the Moon's node, then the values indicated are—

		Obs.	Cal.		Obs.	Cal.
−565	Tables B. $\frac{1}{11}\Delta\beta =$	$-3'\cdot84$	$-3'\cdot84$	Tables C. $\frac{1}{11}\Delta\beta =$	$-3'\cdot23$	$-3'\cdot23$
−172		$-1'\cdot45$	$-2'\cdot47$	$=$	$-1'\cdot05$	$-2'\cdot19$
+132		$-2'\cdot77$	$-1'\cdot70$	$=$	$-3'\cdot07$	$-1'\cdot51$

corresponding to the corrections to the secular acceleration of the node of

$$\text{Tables B. } \Delta B = -4''\cdot25\,(T-1800)^2$$
$$\text{Tables C. } \Delta B = -3''\cdot77\,(T-1800)^2$$

The discordance between the observed and calculated values is very great, so that such a correction can have no real weight. Nor can it be improved by any change in the tabular mean motion of the node.

Taking the values as they stand, they would appear to involve an inequality in the complete expression for the Moon's latitude with a coefficient of about a minute of arc and a period of from seven to eight hundred years. Such a term would undoubtedly reconcile these values of $\Delta\beta$, and render consistent the values found above for the correction to the secular acceleration of the node. But it could not be rendered consistent with the modern observations, for it would mean a change in the Moon's latitude of at least ten to twelve seconds of arc within the period 1820 and 1890; though in all probability the change would be at least two or three times as great, as it is very unlikely that the epoch of the term should be such as to render the amount of change a minimum.

Such a change of the Moon's latitude is quite inconsistent with the observations.

The Arabian Eclipses.

The Arabian eclipses quoted by Professor Newcomb in § 5 of his *Researches on the Motion of the Moon*, from Caussin's edition of "le Livre de la grande Table Hakémite," have to be taken into account to obtain further information on the points which have been discussed in the previous investigation of the Ptolemaic eclipses of the Moon.

Professor Newcomb points out that some of the observed altitudes of the Moon are impossible, and some of the observed altitudes of the clock stars are irreconcilable with each other. Mr Knobel, in *Monthly Notices*, vol. xxxix. pp. 338–340, suggests that these impossible values are due to errors in copying

the Arabian figures, which often differ so slightly that errors can easily be made. I have had no means of referring to the original authorities, but as the result of some inquiries made for me by Mr Marth, I think that the altitude of Arcturus on 925 April 11, which is given as 11°, should be read as 31°; that the altitude of Arcturus on 929 January 27, which is given as 18°, should be 33°; and that the altitude of the Moon on 983 March 1, which is given as 66°, should be 62°.

When the magnitude of the eclipses is given, it does not state whether they were estimated from the upper or lower limb, as is done by Ptolemy, so that when the eclipse is large it is uncertain whether the central line of eclipse passed north or south of the centre of the Sun. This renders the interpretation of the solar eclipses of 993 August 19 and 1004 January 24 somewhat uncertain; and though in each case the most probable path has been taken, some doubt must attach to the results.

The results of the investigation of these early eclipses observed by the Arabian astronomers are as follows:—

The Early Arabian Eclipses:—

Eclipses of the Sun observed at *Bagdad*:—

No.	Date.	Phase.	Tabular Errors. m	Weight.	Magnitude.	Remarks.
1	829 Nov. 29	Beginning	−45·9	0		Rejected.
1a	,,	End	−18·0	1		
2	923 Nov. 10	End	−16·7	1		
3	928 Aug. 17	End	−10·0	2		

Eclipses of the Sun observed at *Cairo*:—

4	977 Dec. 12	Beginning	− 4·5	1	
4a	,,	End	− 3·2	1	
5	978 June 8	Beginning	−19·7	2	
5a	,,	End	− 3·5	2	
6	979 May 28	Beginning	− 6·3	2	
7	985 July 20	Beginning	−24·3	½	0·25
7a	,,	End	−14·7	½	
8	993 Aug. 19	Beginning	− 0·3	2	0·74
8a	,,	End	−19·9	1	
9	1004 Jan. 24	Beginning	−11·1	1	0·02

Eclipses of the Moon observed at *Bagdad*:—

1	854 Aug. 11	Beginning	− 7·1	1
2	856 June 21	Beginning	− 3·7	2
3	923 June 1	End	− 6·7	2
4	925 April 11	Beginning	− 4·5	1
4a	,,	End	+ 2·2	3

Eclipses of the Moon observed at *Bagdad—continued.*

No.	Date.	Phase.	Tabular Errors. m	Weight.	Magnitude.	Remarks.
5	927 Sept. 13	Beginning	+13·2	0	Doubtful. Time possibly calculated.	
6	929 Jan. 27	Beginning	−17·8	1	Altitude corrected for error in copying.	
7	933 Nov. 4	Beginning	− 1·8	2		

Eclipses of the Moon observed at *Cairo* :—

8	979 May 14	End	−11·3	½	Doubtful. Perhaps calculated time.	
9	979 Nov. 6	Beginning	− 3·4	1		
9a	,,	End	−13·9	1		
10	980 May 2	End	− 4·8	1		
11	981 April 21	Beginning	− 2·8	1	Magnitude =0·25.	
11a	,,	End	+ 1·6	1		
12	981 Oct. 15	Beginning	−14·8	1	Magnitude =0·42.	
13	983 March 1	Beginning	− 4·9	1	Altitude corrected for error in copying.	
13a	,,	End	−17·7	2		
14	986 Dec. 18	Beginning	−19·2	2	Magnitude =0·83.	
15	1002 March 1	Beginning	− 2·5	2	Total.	

From these observations are obtained the following expressions :—

Eclipses of the Sun :—

1	$\Delta L = +7'.6$	$+.98\,A_s$	$+.16\,A_c$	$-.06\,B_s$	$+.99\,B_c$	$+9'.70\,\delta l$ *	$-1'.56\,A$
2	= + 6·7	− ·90	+ ·42	− ·10	+ ·99	+8·76	− 1·28
3	= + 4·5	− ·99	+ ·14	+ ·05	+ ·99	+8·72	− 1·26
4	= + 1·7	− ·19	− ·98	− ·11	+ ·99	+8·42	− 1·18
4a	= + 1·1	− ·19	− ·98	− ·11	+ ·99	+8·42	− 1·18
5	= + 5·4	− ·26	+ ·96	+ ·01	− ·99	+8·41	− 1·18
5a	= − 1·9	− ·26	+ ·96	+ ·01	− ·99	+8·41	− 1·18
6	= + 3·8	− ·90	+ ·42	− ·13	− ·99	+8·40	− 1·17
7	= +10·6	+ ·99	− ·14	− ·09	+ ·99	+8·15	− 1·10
7a	= + 9·1	+ ·99	− ·14	− ·09	+ ·99	+8·15	− 1·10
8	= + 0·1	+ ·44	− ·90	− ·04	− ·99	+8·06	− 1·08
8a	= +10·2	+ ·44	− ·90	− ·04	− ·99	+8·06	− 1·08
9	= + 4·4	− ·94	− ·33	− ·05	− ·99	+7·96	− 1·05

Eclipses of the Moon:—

1	$\Delta L = +3'.6$	− ·66	− ·76	− ·01	− ·99	+9·45	− 1·49
2	= + 1·9	+ ·99	+ ·00	+ ·20	− ·98	+9·43	− 1·48
3	= + 3·4	− ·39	+ ·92	+ ·14	+ ·99	+8·77	− 1·28
4	= − 2·3	+ ·93	− ·38	− ·09	+ ·99	+8·75	− 1·27
4a	= + 1·1	+ ·93	− ·38	− ·09	+ ·99	+8·75	− 1·27

Eclipses of the Moon—*continued*

6	= + 9·1	− ·90	− ·44	− ·10	+ ·99	+8·71	− 1.26
7	= + 0·9	− ·57	+ ·82	+ ·02	+ ·99	+8·64	− 1·25
9	= + 1·7	− ·96	− ·27	+ ·11	− ·99	+8·20	− 1·12
9a	= + 7·0	− ·96	− ·27	+ ·11	− ·99	+8·20	− 1·12
10	= + 2·4	+ ·80	+ ·60	+ ·06	+ ·99	+8·20	− 1·11
11	= + 1·4	+ ·07	+ ·99	+ ·02	− ·99	+8·19	− 1·11
11a	= − 0·8	+ ·07	+ ·99	+ ·02	− ·99	+8·19	− 1·11
12	= + 7·7	+ ·37	− ·92	− ·16	− ·99	+8·18	− 1·11
13	= + 2·5	− ·85	− ·53	− ·09	+ ·99	+8·17	− 1·10
13a	= + 8·9	− ·85	− ·53	− ·09	+ ·99	+8·17	− 1·10
14	= + 9·7	+ ·17	+ ·98	− ·10	+ ·99	+8·15	− 1·10
15	= + 1·3	− ·07	− ·99	+ ·06	− ·99	+7·98	− 1·06

From these there are derived the following values :—

			Tables B.—Obs.		Tables C.—Obs.		Weight.	
			ΔL.	ΔE.N.P.D.	ΔL.	ΔE.N.P.D.	ΔL.	ΔE.N.P.D.
Eclipses of the Sun :—								
1	829 Nov.	29	+2′·8	′ ...	+5′·1	′ ...	1	...
2	923 Nov.	10	+3·8	...	+6·0	...	1	...
3	928 Aug.	17	+1·7	...	+3·9	...	2	...
4	977 Dec.	12	−1·4	+2·4	+ ·8	+2·3	2	1
5	978 June	8	− ·8	+4·5	+1·3	+4·6	4	1
6	979 May	28	+1·1	...	+3·3	...	2	...
7	985 July	20	+7·4	+2·2	+9·4	+2·1	1	2
8	993 Aug.	19	+1·0	−4·1*	+3·0	−4·0*	3	1
9	1004 Jan.	24	+2·0	+2·9*	+3·9	+2·8*	1	1
Mean (weighted)			+1·01	+1·68	+3·21	+1·65		
Eclipses of the Moon :—								
1	854 Aug.	11	−0·9	...	+1·3	...	1	...
2	856 June	21	−2·4	...	−0·2	...	2	...
3	923 June	1	+0·5	...	+2·8	...	2	...
4	925 April	11	−2·7	+3·1	−0·5	+3·0	4	1
6	929 Jan.	27	+6·2	...	+8·4	...	1	...
7	933 Nov.	4	−1·9	...	+0·3	..	1	...
9	979 Nov.	6	+1·8	+2·7	+3·9	+2·8	2	1
10	980 May	2	+0·0	...	+2·0	...	1	...
11	981 April	21	−2·1	−0·6	+0·0	−0·7	2	2
12	981 Oct.	15	+5·2	+1·2	+7·2	+1·3	1	1
13	983 March	1	+4·3	+6·2	+6·3	+6·1	3	1
14	986 Dec.	18	+7·3	−1·2	+9·3	−1·3	2	1
15	1002 March	1	−1·2	...	+0·8	..	2	...
Mean (weighted)			+1·15	+1·34	+2·73	+1·50		

The mean results being the same for both solar and lunar eclipses, they may be united into one series and yield the values—

Epoch.	Tables B.—Obs.	Tables C.—Obs.	Weight.	Tables B.—Obs.	Tables C.—Obs.	Weight.
849	$\Delta L = -0'.75$	$= +1'.50$	4	ΔE.N.P.D. =		
927	$= +0'.11$	$= +2'.25$	10	$= +3'.10$	$= +3'.00$	1
981	$= +1'.79$	$= +3'.88$	20	$= +2'.11$	$= +2'.07$	10
999	$= +1'.22$	$= +2'.44$	6	$= -1'.65$	$= -1'.65$	2
Mean (weighted)	$= +1'.23$	$= +3'.21$	36	$= +1'.45$	$= +1'.41$	13

These Arabian eclipses yield results in accord with the theoretical values of the secular acceleration, and admit of being properly represented by no value of the secular acceleration in mean longitude greater than

$$+6''.20$$

whereas it has been shown that the Ptolemaic eclipses of the Moon can be represented by no value less than

$$+7''.40$$

Hence both series cannot be properly represented by any one value of the secular acceleration, as the mean value

$$+6''.80$$

fails to represent either.

Considering the tabular errors in E.N.P.D., there are seven eclipses by which the error can be determined through the observed duration of the eclipse. These are—

Solar Eclipses.	Tables B.—Obs. E.N.P.D.	Tables B.—Obs. $\frac{1}{11}\Delta\beta$	Lunar Eclipses.	Tables B.—Obs. E.N.P.D.	Tables B.—Obs. $\frac{1}{11}\Delta\beta$
977 Dec. 12	$+2'.4$	$-2'.4$	925 April 11	$+3'.1$	$-3'.1$
978 July 8	$+4'.5$	$+4'.5$	979 Nov. 6	$+2'.7$	$+2'.7$
985 July 20	$+2'.2$	$-2'.2$	981 April 21	$+0'.5$	$-0'.5$
			983 March 1	$+6'.2$	$-6'.2$
Mean	$+3'.03$	$-0'.03$		$+3'.12$	$-1'.77$

The errors regarded as errors in E.N.P.D. are the same from both solar and lunar eclipses, but regarded as due to errors in the adopted longitude of the node, they are discordant. If united, they yield the mean values—

$$\Delta\text{E.N.P.D.} = +3'.09 \qquad \tfrac{1}{11}\Delta\beta = -1'.03$$

Hence, like the Ptolemaic eclipses of the Moon, they indicate a uniform correction to the tabular E.N.P.D. rather than the need of a correction to the tabular longitude of the node.

The six eclipses where the recorded magnitude is available for determining the tabular error in E.N.P.D. are—

Solar Eclipses.	Tables B.—Obs. ΔE.N.P.D.	Tables B.—Obs. $\frac{1}{11}\Delta\beta$	Lunar Eclipses.	Tables B.—Obs. ΔE.N.P.D.	Tables B.—Obs. $\frac{1}{11}\Delta\beta$
985 July 20	+2′·2	−2′·2	981 April 21	−1′·8	+1′·8
993 Aug. 19	−4′·1	−4′·1	981 Oct. 15	+1′·2	+1′·2
1004 Jan. 24	+2′·9	+2′·9	986 Dec. 18	−1′·2	+1′·2
Mean	+0′·33	−1′·13		−0′·60	+1′·40

If these be united they yield the mean values—

$$\Delta\text{E.N.P.D.} = -0'\cdot13 \qquad \tfrac{1}{11}\Delta\beta = +0'\cdot13$$

But the separate results are quite discordant, and disagree with the results obtained from the observed durations. A systematic difference between the results obtained from the magnitude and duration of an eclipse is quite possible in the case of these early observations with unassisted vision, especially in the case of a lunar eclipse, but the observations do not indicate any difference of sensible extent. Thus, in the case of the solar eclipse of 985 July 20, the observed duration and magnitude yield the same value for the error in E.N.P.D. In the case of the lunar eclipse of −173 April 30 the observed duration is $2^m\cdot4$ greater than that corresponding to the observed magnitude, whilst in the case of the lunar eclipse of 981 April 21 the observed duration is $4^m\cdot5$ less than that indicated by the magnitude.

If the two series be combined they yield the values

	Table B.—Obs.	Table B.—Obs.	Weight.
Solar Eclipses	ΔE.N.P.D = +1′·68	$\frac{1}{11}\Delta\beta$ = −0′·58	6
Lunar Eclipses	= +1′·53	= −0′·41	7
Mean	ΔE.N.P.D. = +1′·60	$\frac{1}{11}\Delta\beta$ = −0′·49	13

From the preceding investigations the mean values for the errors in E.N.P.D. for the different epochs may be written as

− 565	ΔE.N.P.D. = +3′·50	$\frac{1}{11}\Delta\beta$ = −3′·84
− 172	= +0′·85	= −1′·45
+ 132	= −0′·27	= −2′·77
+ 980	= +1′·60	= −0′·49
+1800	= 0′·00	= 0′·00

It is obvious that no correction to the mean motion in E.N.P.D. nor to the mean motion and secular acceleration of the lunar node will serve to represent these values, though it would be possible to bring them into closer accord by means of a term of long period with a coefficient of about 100″, and a period of seven or eight hundred years.

Disregarding the apparently periodical variations, the observed values are best represented by the expression

$$\tfrac{1}{11}\Delta\beta = -0''\cdot50\,(T-1800) - 0''\cdot33\,(T-1800)^2$$

but the weight of such a correction must be very small—to be, in fact, a mere possibility.

By assuming that the observed tabular error is due to a gradual change in both E.N.P.D. and argument of latitude β, a closer agreement may be obtained. Separating the observations into those made at the two nodes, the results are

Ascending Node.			*Descending Node.*		
−620	ΔE.N.P.D. = −0′·60	weight 2	−540	ΔE.N.P.D. = +5′·14	weight 5
− 2	= −3′·10	2	− 31	= +1′·76	5
+987	= +1′·44	5	+978	= +1′·70	8

These may be regarded as indicating

Obs.		Obs.	Cal.
−580	ΔE.N.P.D. = +2′·27	$\tfrac{1}{11}\Delta\beta$ = −2′·87	−2′·41
− 17	= −0′·67	= −2′·43	−1′·46
+983	= +1′·57	= −0′·13	−0′·20

corresponding to the correction

$$\tfrac{1}{11}\Delta\beta = +1''\cdot00\,(T-1800) - 0''\cdot30\,(T-1800)^2$$

but it is obvious that the results for the middle epoch are discordant.

It is to be remembered that the observations in the three groups were made under somewhat different conditions. The first group consists of observations made by Babylonian astronomers without the aid of any instrumental means; the second group by Grecian astronomers with some instrumental aid; and the third group by Arabian astronomers by the aid of pinule graduated astrolabes. It is possible, therefore, that the estimates of magnitudes may be systematically different; and it is noteworthy that if it be supposed that the Grecian astronomers under-estimated the magnitude of the five eclipses observed between −200 and +136 by a digit as taken from the southern limbs, it would change the observed errors of the middle group to

− 2 ΔE.N.P.D. = −1′·26 −31 ΔE.N.P.D. = +3′·28

corresponding to the derived correction

−17 ΔE.N.P.D. = +1′·01 $\tfrac{1}{11}\beta\Delta$ = +2′·27

thus reducing the discrepancy, but still leaving it far too large for any confidence to be placed in the resulting value for the correction to the node.

How far do the results obtained in the preceding investigations of the observations of these early eclipses agree with Mr Cowell's conclusion that the Moon's argument of latitude shows a secular

acceleration about four and a half seconds greater than that theoretically indicated by the secular decrease in the eccentricity of the terrestrial orbit?

It will be seen that the contact observations of the Ptolemaic eclipses do not indicate the existence of such a correction, and the similar observations of the Arabian eclipses do not extend over a sufficiently long period to enable any conclusion to be formed.

The observed magnitudes of these Ptolemaic eclipses, however, may be regarded as indicating the existence of a secular term of this nature, though of smaller dimensions; but the deduced correction is entitled to little weight, as the separate results are far from accordant unless a considerable term of long period is assumed to exist, and, as they stand, seem more consistent with a secular change in the E.N.P.D. of the Moon.

The magnitude and duration of the early eclipses of the Sun and Moon observed by the Arabian astronomers, likewise, indicate the existence of a correction to the tabular E.N.P.D. of the Moon, which might be ascribed to a secular term in the expression for the Moon's argument of latitude of similar character to that suggested by Mr Cowell; but the separate results show discordances greater than can be ascribed to outstanding errors of observation, which renders it difficult to say whether what is indicated is a change in the adopted secular variation in the argument of latitude, or an analogous change in the E.N.P.D.'s themselves.

When the two series of eclipse observations are united, unfortunately it becomes still less easy to reach a definite conclusion as to which cause the observed deviations from the tabular E.N.P.D. are due to. The observed discordances are so considerable, whatever interpretation be given, that little weight can be assigned to the conclusion which is drawn. Still, though the weight may be small, the observations can be held to support the existence of a secular term in the Moon's argument of latitude greater than can be deduced by theory from the observed decrease in the eccentricity of the terrestrial orbit. Yet they are also consistent, though in a lesser degree, with a secular increase in the observed E.N.P.D., and in this connection it is to be remembered that Tycho Brahe's observations indicate an E.N.P.D. greater by 40″ than that assigned by Hansen's Tables, and that the eclipses of the Sun during the period 1620–1670 indicate an E.N.P.D. from 10″ to 15″ greater than that assigned by the tables.

The only conclusion that can be legitimately drawn from these early eclipse observations appears to be that they are certainly not inconsistent with Mr Cowell's conclusion that the Moon's argument of latitude requires an increased secular acceleration.

In my memoir it was decided that the uncertainty attaching to the correction to the tabular E.N.P.D. indicated by these eclipse observations was too great for them to be taken into account in determining the correction to the tabular value of the longitude of the lunar node.

Natal Observatory:
1906 *August* 17.

On the Early Eclipses. By E. Nevill.

The following notes will serve to elucidate some points in Mr Cowell's paper on the *Ancient Eclipses* in *Monthly Notices*, vol. lxvi. p. 473.

As it was not proposed to use these records of early eclipses of the Sun for deriving corrections to the lunar tables, but only to ascertain how far they were represented by the amended tables, it was judged sufficient to employ approximative forms of calculation; and the data for this purpose were obtained during my visit to England in 1890, from Oppolzer's *Canon der Finsternisse* (Wien, 1887), by applying to his data the corrections necessary to reduce them to the theoretical values of the secular acceleration in mean longitude, longitude of perigee, and longitude of node.

The elements employed by Oppolzer are stated to correspond to the following:—

$$\begin{aligned}
\text{Mean longitude} \quad \zeta &= \text{Hansen's Tables} + \{\ 0''\cdot00 - 26''\cdot34\mathrm{T} - 3''\cdot574\mathrm{T}^2 - 0\cdot004964\mathrm{T}^3\ \} \\
\text{Longitude of Perigee} \quad A &= \text{Hansen's Tables} + \{\ 0\cdot00 - 26\cdot34\mathrm{T} - 16\cdot073\mathrm{T}^2 - 0\cdot016841\mathrm{T}^3\ \} \\
\text{Longitude of Node} \quad B &= \text{Hansen's Tables} + \{\ -5\cdot50 + 53\cdot06\mathrm{T} - 2\cdot956\mathrm{T}^2 - 0\cdot004331\mathrm{T}^3\ \}
\end{aligned}$$

where T denotes centuries reckoned from the epoch 1800.

To reduce these to the theoretical values of the secular accelerations, it is necessary to apply the corrections

$$\begin{aligned}
\text{Mean longitude} &= -2''\cdot496\mathrm{T}^2 \\
\text{Longitude of Perigee} &= +13\cdot128\mathrm{T}^2 \\
\text{Longitude of Node} &= +2\cdot672\mathrm{T}^2
\end{aligned}$$

At the same time, in order to bring the amended tables into accord with modern observations, it is necessary to apply the further corrections

$$\begin{aligned}
\text{Mean longitude} &= -30''\cdot00\mathrm{T} \\
\text{Longitude of Perigee} &= +10\cdot80\mathrm{T} \\
\text{Longitude of Node} &= +5\cdot00\mathrm{T}
\end{aligned}$$

Hence the total corrections to be applied to the elements of the Moon's orbit employed by Oppolzer in order to reduce them to the theoretical values are:

$$\begin{aligned}
\Delta\zeta &= +0''\cdot00 - 3''\cdot66\mathrm{T} - 2''\cdot496\mathrm{T}^2 \\
\Delta A &= +0\cdot00 + 37\cdot14\mathrm{T} + 13\cdot128\mathrm{T}^2 \\
\Delta B &= +0\cdot00 - 48\cdot06\mathrm{T} + 2\cdot672\mathrm{T}^2
\end{aligned}$$

Oppolzer's formulæ for calculating the latitude ϕ and longitude λ of the position of the curve of central totality for the beginning, middle, and end of an eclipse are

$$\begin{aligned}
&\text{Beginning} \quad \sin\phi = -\cos\delta'\cos(\mathrm{N}'+\mathrm{W}) \qquad \lambda = -\mu + \frac{15^\circ}{n}\cos\mathrm{W} - \tan(\mathrm{N}'+\mathrm{W})\operatorname{cosec}\delta' \\
&\text{Middle} \quad \sin(\phi-\delta') = \sin\mathrm{W}\operatorname{cosec}\mathrm{N}' \qquad \lambda = -\mu - \frac{15^\circ}{n}\cot\mathrm{N}'\sin\mathrm{W} \\
&\text{End} \quad \sin\phi = +\cos\delta'\cos(\mathrm{N}'-\mathrm{W}) \qquad \lambda = -\mu - \frac{15^\circ}{n}\cos\mathrm{W} - \tan(\mathrm{N}'-\mathrm{W})\operatorname{cosec}\delta'
\end{aligned}$$

where

$$\sin W = \frac{\sin(b-b')}{\sin(\pi-\pi')} \cdot \sin(N'+h)$$

$$\tan h = \tan \epsilon \cdot \cos L'$$

$$\mu = 15^\circ.H - \frac{15^\circ}{n} \cdot \sin W \cdot \cot(N'+h) - \text{Equation of Time.}$$

Here

H denotes the epoch of conjunction measured from Greenwich Noon.
b, b' denote the latitude of the Moon and Sun.
π, π' denote the horizontal parallaxes of the Moon and Sun.
L', δ' denote the true longitude and declination of the Sun.
ϵ denotes the obliquity of the Ecliptic.

The other quantities are taken as they stand from Oppolzer's *Canon*.

It is obvious that it is only W and μ that will suffer any material change by the introduction of the correction $\Delta\zeta$, ΔA, and ΔB.

Then α denoting the Moon's mean anomaly and β the Moon's argument of latitude,

$$\Delta \sin W = \Delta B \cdot \cos(b-b') \operatorname{cosec}(\pi-\pi') \sin(N'+h)$$

where approximately

$$\Delta B = -\cos\beta\{(0\cdot092 - 0\cdot006\cos\alpha)\Delta B + 0\cdot007\Delta\zeta\}$$

and

$$\Delta\mu = 15^\circ\{\Delta H - \Delta B \cdot n^{-1} \cdot \cos(N'+h)\}$$

where approximately

$$\Delta\mu = -(28^\circ\cdot65 - 1^\circ\cdot38\cos\alpha)\Delta\zeta + 2^\circ\cdot64\cos\alpha \cdot \Delta A + 0^\circ\cdot25\Delta B$$

The changed path of the curve of central totality can be deduced from the comparison of the new positions of the beginning, middle, and the end of eclipse with those given by Oppolzer.

For the required purpose this approximation was sufficiently accurate.

It was in the same manner that the approximate paths of the different eclipses were calculated on Mr Cowell's original data. In the case of the eclipse of -1116 June 18 an accidental error in copying out the final correction as plus instead of minus led to an inaccurate result, and this eclipse cannot have been that observed at Babylon.

I cannot concur with Mr Cowell's method of bringing -1062 July 31 within the month of *Sivan*. The intercalary months were never inserted at the beginning of the year, and the first of *Nisan* could not have been later than the early part of April, so that the month of *Sivan* must have ended in the beginning of July at the latest. Nor can I concur with Mr Maunder in supposing that though July 31st could not have fallen within *Sivan* according to the known Babylonian calendar, yet that it may be legitimately assumed that, as the epoch is prior to the era of *Nabonasser*, the calendar may be supposed to be quite different from that known to

be used after that era. If such arbitrary assumptions have to be made, the records of these eclipses are absolutely valueless.

On this question I would observe, merely, that it must not be forgotten that for many centuries prior to the epoch of this eclipse the Babylonians had possessed a well-regulated definite calendar, sufficiently certain and predicate to enable its being incorporated in deeds, leases, and commercial agreements. For centuries the beginning of the year was fixed by the occurrence of the equinox, and it is most unlikely that they were dependent for the fixing of this event on the observation of actual New Moon, or on the heliacal or other fancy rising or setting of a star, when, in the passage of the shadow from the Sun over a fixed line on the Temple floor or courtyard, the line midway between the longest and shortest shadow during the year, they had a means of fixing the date of the equinox to the very day. Then, when the equinox fell after the first half of the fixed month, the year was made full by the insertion of an intercalary month at the end of the year. The reckoning of time in this manner by the dimensions of the shadows occurs in the early history of all races, and is universal in the East even now. The heliacal rising and setting of stars and the visibility of the new moon would have their religious and astrological signification, but so clumsy and indirect a method of fixing the equinox would not have been used when so much easier a method was available. Recently, every year has brought further evidence, in the shape of astronomical records and calculations, that the Assyrians, Babylonians, and even the early Chaldeans possessed a much better knowledge of astronomy and much better means of making astronomical observations than had been assigned to them, and that they could measure epochs and intervals of time with some certainty, probably by the equivalent to a dial on the floor of the Temple or its courtyard, as well as by contrivances of the nature of clepsammia or clepsydra. This, however, is beyond the present subject.

Fuller discussion of the eclipse seen at Babylon "on the 26th *Sivan*" is useless until the full details of the record are before us, and it is possible to critically examine the reasons for assigning it to some date in the eleventh century before our era, for at present there exist no data for estimating its value as the record of an eclipse at any place or at any epoch. If the record can be established as proving the occurrence of a total eclipse visible at Babylon in the eleventh century before our era, it is undoubtedly of the very highest importance as fixing both astronomical and chronological elements, but we must have the record with all ancillary evidence before its true bearings can be properly discussed.

Mr Cowell points out that the failure of the observations of the Sun during the period 1750–1900 to show any signs of the assumed secular acceleration in its mean longitude may be due to the existence of an unknown term of very long period. That is indubitable; and on that basis it must be admitted that these

observations of the Sun cannot be said to be inconsistent with the extended hypothesis.

The evidence that has been advanced serves to show that the facts brought forward by Mr Cowell in support of his views can be explained without having recourse to a new secular acceleration in the mean motion of the Earth, so that these facts do not suffice to establish the existence of such a secular acceleration. But if this is all, they certainly do not suffice to disprove the existence of such a term. If that is to be done, it must be by further direct evidence, to which I have no access, though it may exist. During past years in my researches I have repeatedly found evidence of an unexplained apparent secular acceleration in the motion of the Moon's argument of latitude, but I have doubted its reality, as no explanation of its origin seemed available. The explanation advanced by Mr Cowell, that it might be due to a secular acceleration in the mean motion of the Earth, did not occur to me, or I should have taken the necessary steps to obtain the data to investigate the matter. This, I presume, will be done by Mr Cowell. I do not see any theoretical explanation of such a secular acceleration which does not seem to involve great difficulties. Mr Cowell's tentative explanation (*Monthly Notices*, vol. lxvi. p. 352) I think he will find untenable on review.

Natal Observatory:
1906 *August* 23.

The Mediæval Eclipses of Celoria. By P. H. Cowell.

Professor Celoria has collected a number of references to the total eclipses of 1239 June 3 and 1241 October 6. He considers that the former eclipse was certainly total at Piacenza and at Lesina, and that the latter was total at Stadt and at Ellwangen. He draws the limits of totality according to Hansen's Tables (without Newcomb's corrections), and concludes that these tables are in serious error at the epoch of the two eclipses.

Professor Celoria's diagrams show that the conditions that he imposes are very stringent ones. The zone of totality for the 1239 eclipse is to be displaced not less than enough to secure totality at Piacenza, and not so much as to destroy totality at Lesina, and these limits are very narrow. They become still narrower if it be remembered that Hansen's semi-diameter is considerably too large for eclipse purposes. A similar remark applies to the eclipse of 1241. Professor Celoria's assumptions, therefore, lead to two equations of condition, one for each eclipse, in which the margin of uncertainty is very slight. The combination of the two should therefore give an excellent determination of the corrections required by the mean elongation and by the argument of latitude.

The form of the equations of condition can be most simply found. Let k denote, as in my previous papers on eclipses, the

ratio of the Moon's motion in latitude to the difference of motion in longitude as seen by the observer, and let $\delta(V - V')$ and δU be the corrections required by the difference of longitude and by the Moon's latitude. Then the equation of condition must be

$$\delta U - k\delta(V - V') = a$$

where a is a constant, that depends upon the tables to be corrected.

That this is the true equation of condition is most easily seen; for if δU, $\delta(V - V')$ be one solution, $\delta U + k\delta t$, $\delta(V - V') + \delta t$ must be another solution, corresponding to a small change in the assumed time at which the eclipse occurred.

For any place on the Earth's axis, including the centre of the Earth and the two poles, the parallactic displacements are constants, and the value of k will be $\pm 0{\cdot}10$, the upper sign corresponding to the ascending node. For moderate latitudes k is approximately

$$\pm\ 0{\cdot}14 + 0{\cdot}14 \cos h$$

where h is the sidereal time.

Of course this formula is only a rough one, but it serves the purpose, before undertaking laborious calculations, of predicting that the equations of condition will involve

$$\left.\begin{array}{r}\delta U - 0{\cdot}12\ \delta(V - V') \text{ for the 1239 eclipse} \\ \text{and } \delta U + 0{\cdot}26\ \delta(V - V') \text{ for the 1241 eclipse.}\end{array}\right.$$

Writing

$$\begin{aligned}\delta U &= \pm 0{\cdot}09\ \delta F \\ \delta\,(V - V') &= \delta D\end{aligned}$$

the unknown corrections to the argument of latitude and mean elongation enter in the form

$$\begin{aligned}&0{\cdot}09\ \delta F - 0{\cdot}12\ \delta D \\ \text{and}\quad &-0{\cdot}09\ \delta F + 0{\cdot}26\ \delta D\end{aligned}$$

We shall not therefore have any trouble arising from the determinant of the coefficients being small.

Professor Celoria has obtained the equations

$$\begin{aligned}9{\cdot}8635\ \delta v - 5{\cdot}4914\ \delta\Omega &= \ \ 35'{\cdot}40 \\ -57{\cdot}7624\ \delta v - 1{\cdot}4098\ \delta\Omega &= -42{\cdot}60\end{aligned}$$

These equations cannot be reconciled with the form of mine, which has been justified by such elementary reasoning.

I obtain the following expressions for the difference of apparent latitude at the instant of apparent conjunction in longitude, δF, δD being corrections to the values of F, D that correspond to the formulæ in *Monthly Notices*, vol. lxvi., p. 525.

1239, for Piacenza	$0{\cdot}13\delta D - 0{\cdot}09\delta F + 89''$
1239, for Lesina	$0{\cdot}11\delta D - 0{\cdot}09\delta F - 47''$
1241, for Stadt	$-0{\cdot}25\delta D + 0{\cdot}09\delta F + 29''$
1241, for Ellwangen	$-0{\cdot}27\delta D + 0{\cdot}09\delta F - 57''$

Hence, in order to produce totality at all four places, we must have

$$0{\cdot}12\delta D - 0{\cdot}09\delta F = -21''$$

and

$$-0{\cdot}26\delta D + 0{\cdot}09\delta F = +14''$$

with errors of not more than three or four seconds on the right-hand sides.

The solution of these equations is $\delta D = 50''$, $\delta F = +300''$.

The value of δD is a possible one, but the value of δF is utterly impossible. It cannot possibly be reconciled with modern observations. Matters are even worse if the comparison be made with the present tables. The latter require an even larger correction δF.

I interpret allusions to darkness and to the appearance of stars to mean that the place of observation was not necessarily within the zone of totality, but within a somewhat wider zone, within which not more than about twenty seconds of the Sun's diameter was visible.

Now this explanation not only serves to dispose of an inconveniently large correction to the formulæ used in calculation, but it is absolutely required by the records. For at Altenzelle in 1241 we have the record " . . . . et stellæ apparuerunt," and at Vienna we have two records to the same effect. Professor Celoria confines the places to which he assigns black rings on his maps to within a zone not wider than the zone of totality. As the distance between Ellwangen and Vienna is about as great, when projected perpendicularly to the central line, as the greatest width that can be ascribed to the zone within which stars appeared, the central line probably was about equidistant from the two places. This fits my formulæ well, but I cannot use the principle to discriminate between my formulæ and the present tables.

In 1239 the record at Milan has to be rejected, as Professor Celoria suggests, on the ground that the writer lived 160 years later. We are then left with the records that stars were seen at Piacenza, and that the eclipse was certainly total at Lesina.

To sum up, therefore, my formula cannot be accepted unless it be admitted that at Piacenza in 1239 stars might have been seen with about 20″ of the Sun's diameter visible; on the other hand, if the theoretical secular acceleration be attributed to the argument of latitude, 30″ of the Sun's diameter would have been visible. I am not prepared to say that 20″ is possible and 30″ impossible, and therefore no proof can be based upon these eclipses. I feel bound, however, to examine all evidence that I come across, and I can only say that I am perfectly satisfied with the way in which my formulæ stand the test in the present instances.

The outline of the calculations is as follows:—

Changes in a Julian century $\times 10^{-7}$.

	1239	1241
	″	″
True Longitude of Moon .	198·64	199·72
„ „ Sun .	12·53	13·12
True Elongation . . .	186·11	186·60

	1239 Piacenza.	1239 Lesina.	1241 Stadt.	1241 Ellwangen.
Parallax in Longitude	55′3	55′2	45′0	49′5
Apparent Elongation	130°8	130°9	141°6	137°1
Geocentric Latitude of Moon		+18′42		−18′24
Parallax in Latitude	−1′2	−4′2	−17′7	−19′6
Apparent Latitude of Moon	+17′2	+14′2	−35′9	−37′8
Hence k	+0·13	+0·11	−0·25	−0·27

	1239	1241
T in centuries from 1800 Jan. 0·06 M T	−5·6054754	−5·5820399
G.M.T. in degrees	+3°·9654	−2°·6451

F	5	23′	36″·4	169°	28′	27″·5
D	2	0	1·6	357	3	26·9
g	344	26	51·5	7	50	14·0
g'	168	39	16·1	292	18	35·7
$-\Omega$	284	51	39·0	330	11	20·6

Inequalities of Moon's Longitude :—		
α. 21 largest terms, coefficients over 20″	−5096″·2	+3299″·2
β. 30 next largest solar terms, coefficients over 1″·35	−5·9	+53·5
γ. 58 next largest solar terms, coefficients over 0″·15	+4·0	−0·4
δ. Figure of Earth terms	+7·1	+3·2
Inequalities of Moon's Latitude :—		
α. 14 largest terms, coefficients over 10″	+1271·3	+3118·0
β. 42 next largest solar terms, coefficients over 0″·55	−21·1	−63·7
γ. 62 next largest solar terms, coefficients over 0″·06	−0·4	−3·9
δ. Figure of Earth terms	−7·5	+1·2
Moon's sine parallax. Constant, 3422″·7 :—		
α. 5 largest terms, coefficients over 10″	+3671·5	+3678·7
β. 9 next largest solar terms, coefficients over 0″·55	−0·9	+4·2
γ. 17 next largest solar terms, coefficients over 0″·06	+1·0	−0·2
Inequalities of Sun's Longitude :—		
Three terms of equation of centre	+22′ 33″·1	−1° 49′ 7″·7

	Piacenza.	Lesina.	Stadt.	Ellwangen.
Geocentric Latitude	44°51′·0	42°58′·6	53°24′·6	48°45′·6
Radius Vector ρ	·99830	·99841	·99779	·99807
Longitude, East of Greenwich	9°41′·55	16°28′·22	9°28′·43	10°9′·92

Parallax in Longitude :—				
α. Terms of first order	−740″·3	−1055″·0	+850″·1	+726″·7
β. „ second „	−11·9	−16·8	+7·1	+6·9
γ. „ third „	−0·2	−0·2	+0·2	+0·2

Parallax in Latitude :—

α. Terms of first order $-1329\cdot8$ $-1230\cdot1$ $-3122\cdot2$ $-3006\cdot7$

β. Smaller terms $-\ 1\cdot4$ $+\ 0\cdot2$ $-\ 0\cdot7$ $+\ 0\cdot3$

The formulæ used for the arguments are those given in *M. N.*, lxvi. p. 525.

The inequalities of the Moon's Longitude, Latitude, and Parallax are taken from Brown, *M. N.*, lxv. p. 276.

The inequalities of the Sun's Longitude and the obliquity of the ecliptic are taken from Newcomb, Tables of the Sun.

The figure of Earth's terms used are :—

For Moon's Longitude $+ 6''\cdot6 \sin\Omega - 0''\cdot7 \cos\Omega + 1''\cdot0 \sin\Omega \cos$

For Moon's Latitude $- 8''\cdot1 \sin L + 1''\cdot5 \cos L - 1''\cdot0 \sin g \cos L - 0''\cdot3 \sin(F - \Omega)$

Nutation is not applied, since it is the same for both Sun and Moon, and not sufficiently important in its effect on the parallax.

The planetary inequalities have not been calculated for either Sun or Moon.

The formulæ used for parallax are

$$q = \rho\,(\sin p - p')$$

Parallax in longitude $- bq - abq^2 + (\frac{1}{3}b^3 - a^2b)q^3 - \frac{1}{2}bqU^2$

" latitude $- cq + aq(U - cq) - \frac{1}{3}U^3$

where $a = \cos\lambda\cos h\cos V + \cos\lambda\sin h\cos\epsilon\sin V + \sin\lambda\sin\epsilon\sin V$

$b = -\cos\lambda\cos h\sin V + \cos\lambda\sin h\cos\epsilon\cos V + \sin\lambda\sin\epsilon\cos V$

$c = -\cos\lambda\sin h\sin\epsilon + \sin\lambda\cos\epsilon$

Less exact calculations would have been sufficient if I could have foreseen that the only result is that my formulæ expose less of the Sun's disc at the places named as places where stars appeared than the theoretical formulæ.

The present paper is most easily read in connection with Professor Celoria's diagrams. These, however, can be sufficiently reproduced from the following extracts from Professor Celoria's papers. The latitudes are geographical.

1239 June 3. Limits of Totality.

North		South	
43° 6′·84 N	0° 29′·79 E	40° 49′·52 N	1° 12′·58 E
44 18·97	8 23·34	41 58·58	8 33·51
44 43·80	17 1·14	42 24·64	16 34·23

1241 October 6. Limits of Totality.

North		South	
58° 35′·96 N	6° 42′·01 E	55° 56′·48 N	4° 59′·05 E
51 57·27	12 48·14	49 41·56	10 49·69
46 37·08	17 24·99	44 34·64	15 20·49

and

Place.	Latitude.	Longitude.
Milan	45° 27′ 59″	9° 11′ 14″
Altenzelle	51 3′·55	13 16′·30
Vienna	48 12·55	16 22·52

The positions of the other places mentioned in this paper are given above.

Professor Celoria's extracts from mediæval records represent a vast amount of valuable labour. It is advisable, however, to warn readers that his figures for the 1239 eclipse must not be taken from his original paper, but from his own revised figures in his second paper.

The Irregular Movement of the Earth's Axis of Rotation: a Contribution towards the Analysis of its Causes. By Professor J. Larmor, F.R.S., and Major E. H. Hills, C.M.G.*

Much material, defining with increasing accuracy the irregular wanderings of the Earth's axis of rotation, has now been accumulating for a long series of years. The attempt to decompose the movement into regular harmonic components excited interest some six or eight years ago. Since that time the more systematic data obtained and analysed by the International Organisation have given greater precision to the path of the Pole; and, while a definite astronomical discussion must rest with the experts, curiosity as to the general physical causes of the phenomenon is legitimate. It has long been recognised that displacement of material on the Earth's surface due to meteorological changes (melting of polar ice, long-period barometric fluctuations, etc.) must be a prominent agent, and may indeed be taken to be the main one; while Newcomb has pointed out that the free Eulerian oscillatory period must be very different, for a nearly spherically balanced Earth, if it is elastically deformable under the centrifugal force, from what it would be if it were rigid, thus accounting for the unexpected value of the Chandler period.

It is shown below that, without making any hypothesis except the natural one, that this free precessional period is fixed in duration and determines the average duration of the revolutions of the Pole of rotation, it is easy by a graphical process to deduce from the path of the pole a map of the varying torque which must be acting in order to produce that path, and thence to infer as to the character of the displacements of terrestrial material that must be taking place in order to originate that torque on the Earth as a whole. It has seemed worth while to carry this out in a preliminary way. It has also been thought worth while to set down various dynamical considerations which may prove useful in a systematic analysis of the observational results.

Let ω_1, ω_2, Ω be the component angular velocities of the Earth referred to axes moving with itself, the latter being around the axis of figure. Thus $\omega_1/\Omega, \omega_2/\Omega$ are the angular co-ordinates of the pole of rotation measured on the Earth's surface, and represent

* Read in part at the British Association, August 1906.

directly the change of latitude; they are different from the absolute co-ordinates of the pole on the celestial sphere, in the order of the ratio C/(C – A), as appears from the Poinsot geometrical representation of the free precessional motion, C and A being the *effective* polar and equatorial moments of inertia.* The dynamical equations of the Earth's free precession are, with sufficient accuracy, in terms of the angular momentum (h_1, h_2, H),

$$\frac{dh_1}{dt} - h_2\Omega + H\omega_2 = 0$$

$$\frac{dh_2}{dt} - H\omega_1 + h_1\Omega = 0,$$

where, D, E, F being products of inertia,

$$h_1 = A\omega_1 - F\omega_2 - E\Omega, \quad h_2 = B\omega_2 - D\Omega - F\omega_1, \quad H = C\Omega - D\omega_2 - E\omega_1$$

Thus

$$A\dot{\omega}_1 - (C - A)\Omega\omega_2 = -L'$$
$$A\dot{\omega}_2 + (C - A)\Omega\omega_1 = -M',$$

where Ω may be taken constant, because its variation would multiply in these equations two factors, each small of the first order; and where L′, M′ include, in addition to the kinetic forcives due to the location of mobile material attached to the earth, the terms

$$\dot{A}\omega_1 - \dot{F}\omega_2 - \dot{E}\Omega, \quad \dot{B}\omega_2 - \dot{D}\Omega - \dot{F}\omega_1$$

Here the terms on the left would be rates of change of angular momentum if the configuration remained constant. The terms on the right include the reversed rates of change of the same, taking the angular velocity constant but the configuration altering. To determine these latter terms, as arising from the reaction of mobile terrestrial material, we can consider them directly as representing the centrifugal forces of this loose material, together with the reversed gradient of the angular momentum, due to the Earth's rotation, of new negative material in the original position and of new positive material in the altered position.

The difference between this procedure and the usual one adopted by previous writers † is that the investigation of the slight

* *Cf.* Routh, *Dynamics*, ii. §§ 180–2, 533. On account of the largeness of this factor C/(C – A) the axis of rotation is practically fixed *in space*, except in so far as the extraneous attractions of the Sun and Moon operate in causing forced precession, so that the effect here in question is simply a change of latitude. *Cf.* Maxwell [" On a Dynamical Top . . . . with some Suggestions as to the Earth's Motion," *Trans. R. S. Edin.*, 1857, *Scientific Papers* i. (pp. 259–61)], who appears to have been the first to apply the Euleriar theory precisely to the Earth, and to examine the Greenwich observations in search of a 306-day period in latitude. According to Professor Newcomb, *Monthly Notices*, lii., 1892, pp. 336–41, searches for terms in the latitude, of this period, instituted at Pulkowa by Peters, and at Washington, in 1862–7, had already yielded negative results. In Newcomb's paper, which pointed out and estimated roughly the increase of free period produced by elastic yielding, the influence of irregular meteorological displacements was also considered.

† For an account with references, *cf.* Routh, *Dynamics*, ii. §§ 24, 533–5.

changes of position of the Earth's principal axes of inertia arising from displacement of material is evaded, by considering an unchanging Earth, with effective moments of inertia (A, A, C), which is subject to force arising from the kinetic reaction exerted on it by this additional and independent material, moving over it and at the same time maintained by it in diurnal rotation.

The effect of centrifugal force, in flattening elastically the terrestrial spheroid, simply modifies its effective or dynamical moments of inertia according to the principle arrived at in a previous discussion,* viz. that the moments of inertia which would exist in the absence of diurnal rotation, but on the assumption that the Earth's form when centrifugal force is thus removed is determined by a linear law of elasticity, are to be employed in dynamical investigations which take account of the elastic yielding of the Earth. There would be consequently an increase in the free precessional period (actually as observed it is from 306 to about 428 days) in the manner first pointed out by Newcomb.

Refer now the problem thus formulated to axes of ω_1 and ω_2 rotating with angular velocity $\Omega(C-A)/A$, that of the undisturbed

* "On the Earth's Free Eulerian Precession," *Proc. Camb. Phil. Soc.*, May 25, 1896, p. 186.

The argument there employed, briefly stated in more analytic form, is as follows. Let A′, B′, C′ be the principal moments of inertia of the Earth when unstrained by centrifugal force; and let I be the change of moment of inertia round its own axis, due to the equatorial protuberance raised by that force. This axis is in the direction of the resultant angular velocity $(\omega_1, \omega_2, \omega_3)$; and it is implied that it is very near to the principal axis of greatest moment C′, so that $\omega_3(=\Omega)$ is practically constant and great compared with ω_1 and ω_2. It is involved also in this restriction that I is a constant up to the first power of ω_1/Ω or ω_2/Ω. Referred to the principal axes, the total component angular momenta are

$$h_1=A'\omega_1+I\omega_1,\ h_2=B'\omega_2+I\omega_2,\ h_3=C'\omega_3+I\omega_3$$

The equations of motion referred to the rotating axes are of the well-known vector type

$$\dot{h}_1-h_2\omega_3+h_3\omega_2=L$$

When A and B are equal, the third of them is

$$\frac{d}{dt}(C\omega_3)=N$$

where C is the effective moment of inertia C′+I: when N is null ω_3 is thus constant, say Ω, up to the first order. The other two equations are

$$\frac{d}{dt}\cdot(A'+I)\omega_1+(C'-B')\Omega\omega=L_2$$

$$\frac{d}{dt}\cdot(B'+I)\omega_2-(C'-A')\Omega\omega_1=M,$$

which in the case of approximate symmetry involve a free period $2\pi(A'+I)/(C'-A')\Omega$, and similarly in the general case, thus depending only on A′, B′, C′ when I is small.

The present procedure absorbs the effect of this regular change of form due to strain into modified moments of inertia, while it sets out the effect of erratic displacement of (additional) material on the rotating Earth as a kinetic forcive.

free Eulerian precession, viz., about 428 days. The equations of movement assume the form

$$A\dot{\omega}_1 = -L', \qquad A\dot{\omega}_2 = -M',$$

the same as if the axes were fixed and there were no diurnal rotation; that is, the polar axis moves in the earth, relative to these rotating axes of co-ordinates, along the direction of the reversed resultant of the torques L′ and M′.

It seems useful, therefore, to plot the course of the Pole relative to co-ordinate axes rotating with the mean Chandler period, marking, at intervals along the curve, both the time, and the longitude of one of the revolving axes of co-ordinates at that time; for the velocity along the curve at any instant will then give the direction and magnitude of that part of the rate of change of (L′, M′), the transverse component of the centrifugal torque of the loose material and the time-gradient of the angular momentum arising from transport of this material, when the velocity of rotation is imagined unaltered. Such change can therefore be partially located, as *infra*; if it is mainly due to displacements of surface-material, of thermal or meteorological type, it should show seasonal recurrences, and may prove to be in part due to slight change in oceanic or barometric levels.

Aggregate rough estimates of mere order of magnitude are easiest made directly, without use of these rotating axes. Thus a surface depression of 1 foot over a square mile, extending down in gradually diminishing amount to 30 miles, would involve an effective displacement of a layer 1 foot thick through 15 miles downward. In latitude 45°, where the effect in this respect would be greatest, this displacement would change the resultant transverse component of angular momentum of the earth by $4000\ \Omega \frac{2\frac{3}{4}\cdot 15}{5280} \cos^2 45°$, the whole angular momentum of the earth being in the same units Ω. $5\frac{1}{2}\cdot \frac{4}{3}\pi(4000)^3\cdot \frac{2}{5}(4000)^2$; this takes the density of surface material to be $2\frac{3}{4}$ and that of the whole Earth $5\frac{1}{2}$. The polar axis would thereby be displaced through an angle equal in absolute measure to the ratio of these quantities; in seconds of arc it would be about 3.10^{-18}. Thus local displacements by earthquakes can have no sensible direct effect on motion of the Pole. But more important is the centrifugal effect representable by displacement of the axis of inertia of the Earth, round which the free precession of the polar axis is taking place. This angular displacement is $h/(C-A)$, when h is the product of inertia thus introduced: this is of $C/(C-A)$ times (about 300 times) the order of the direct effect on the Pole of rotation, so that in the present way of viewing the matter the changing origin of precession is practically everything.* It has been found, however, by Milne (Bakerian Lecture, *Roy. Soc. Proc.*, 1906), that, for the

* Sir G. H. Darwin has recently expressed the opinion that on the whole the effect of earthquakes may be to bring the axis of rotation nearer to the axis of figure, and thus damp the polar movement.

small range of time (two years) then investigated by him, sharp curvatures in the polar movement appeared to be on the whole concomitant with earthquakes; the latter may be promoted perhaps by the changes of superficial or internal loading along meridians, that are the main cause of the irregular motion of the Pole and are greatest when the curvature of its path is sharpest. The procedure indicated in this note would locate to some extent this displacement or change of loading, and thus test that theory.*

The effect of transfer of water from the Poles towards mean latitudes, arising from melting of Arctic ice, may be estimated either by considering an added layer, of thickness positive or negative according to the locality, and of null aggregate amount spread over the whole ocean, or by estimating directly as above the change of angular momentum involved in the displacement of each portion of the material. A displacement of the Pole of rotation in the Earth in a given direction, when referred to the rotating axes as above, would imply alteration of intrinsic angular momentum of surface load in the neighbourhood of the meridian circle containing that direction, which would be a defect in the northern quadrant in front of that direction or an increase in the northern quadrant behind it, and *vice versa* for the southern quadrants. Thus water rapidly moved from the Poles, where it has little angular momentum, so as to cover to a depth of 1 foot a region 4000 miles square, in middle latitudes, would displace the Pole of rotation in the Earth by something of the order of 2 seconds of arc; for it would involve a new transverse angular momentum $\Omega \cos^2 45^\circ \, 4000^5/5280$ in the same units as above. It is readily seen that the principal axis of inertia, about which the free precession would continue, would be displaced in the opposite direction through an angle of the same order.

In reducing the International Observations of change of latitude at the selected observatories extending round the Earth, it has been found necessary to include a change common to all longitudes, which at first sight could only arise from change of form of the spheroid which represents the terrestrial sea-level. This might be in part due to gravitational influence of the displaced material; yet the removal by melting of 10 feet of ice over a polar area 500 miles in diameter would produce a change of attraction which could not, in middle latitudes, raise the Pole by more than

* The case here considered is 15 cubic miles of material displaced vertically 1 foot. Professor Milne informs us that the result of an actual earthquake might be 10,000,000 cubic miles displaced vertically or horizontally through 10 feet. This would multiply the figure in the text by 7.10^6, thus giving 2.10^{-6} seconds of arc. After this sudden shift of the axis of rotation in the Earth, the free precession would continue, but it would be around a new principal axis of inertia displaced from the original one by a quantity of the same small order of magnitude multiplied by C/(C − A), that is by 300, giving a result of the order 10^{-3} seconds.

Sir G. H. Darwin has estimated (*Phil. Trans.*, 1876) that $\frac{1}{170}$ of the area of Africa rising or falling *in situ* through 10 feet would produce 0·2 seconds of change, the rising or sinking being presumably taken as not merely superficial.

1/1000 of the order of magnitude required. As the effect seems to be real, some indirect, perhaps seasonal instrumental, cause must apparently be sought for.

In trying, as here, to separate out the meteorological displacements of the Pole from the true free precession which would in their absence be a regular circular motion, by referring the whole to rotating axes, the essential point is to assign as correctly as possible the period of this precession, for that determines the velocity of rotation to be given to the axes of co-ordinates. Elastic yielding of the Earth will prolong the period beyond the 306 days that would belong to a rigid solid. Hough has shown * that an average modulus of rigidity of a solid Earth even so great as that of steel would involve the prolongation of the period to the above value, 428 days, which represents the periodicity of the observed path of the Pole. Now whatever be the cause, elastic or fluid displacement, or both, that thus alters the effective dynamical moments of inertia of the Earth, it may be presumed that it alters them to a constant extent over a fairly long period of time. Thus the true free, or Eulerian, precession would maintain a rotation of the Pole fairly constant, while the meteorological disturbance superposed on it would in the long-run have no rotational quality one way round or the other. In applying the method of analysis of the complex motion that is here proposed, the angular velocity of true precession may therefore be obtained by taking the mean of the observed times of revolution of the Pole, as Chandler originally pointed out.

The Eulerian principal axes fixed in the Earth, or still better the axes rotating as specified above, are appropriate to the analysis of the effects of moments or torques, which arise from change of loading and are therefore themselves revolving on the whole with the Earth. On the other hand, torques which only slowly change in direction in space, such as those arising from the attraction of the Sun and Moon on the protuberant parts of the oblate terrestrial spheroid, are most amenable to dynamical analysis when referred to fixed axes. They produce mainly the ordinary forced astronomical precessions and nutations, on which the varying elastic yielding of the Earth has no sensible kinetic influence; while the intensity of the torque of attraction depends on the instantaneous geometrical value of $C - A$, as determined by distribution alone. The distribution of mass thus governs the solar and lunar precessions.† These forced precessions depend on internal fluidity,

* *Phil. Trans.*, 1895. It is (as above) the near approach to sphericity that makes so slight a yielding to the changes of centrifugal and other stresses effective in this manner.

† There is one case, however, in which, as the equations of motion given above show, a small extraneous forcive would cause a wandering of the Pole in the Earth itself, much greater than its change of direction in space, *i.e.* than the astronomical precession thereby caused, that, namely, of a forcive having a period in longitude relative to the Earth's rotation, of the order of the free period of 428 days. Such a term in the forcive would cause a forced

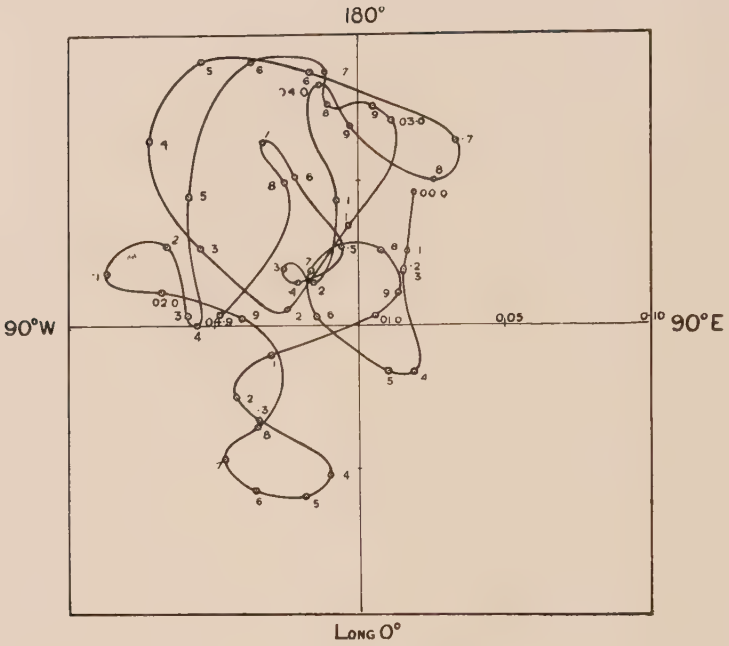
etc., only in so far as it modifies the effective inertia-moment C or A; while, on the other hand, the free precession depends on the effective or kinetic value of the small difference (C – A)/C.

The tides would be about the same at antipodal points if contours of the land were symmetrical, and the two opposite tidal protuberances would be additive in their effect on the free precessional moment. At first sight, it might appear that the estimate given above for a local overflow of a foot of water from the Polar regions would involve that, even apart from the irregularity of form of the oceans, the features of the various tidal components travelling round the Earth must be reproduced to some extent in an exact diagram of the torque, owing to the tidal flow demanding alteration of the angular momentum of the water relative to the Earth's rotation, and to its centrifugal force. But in so far as it is the Earth that turns round under the nearly stationary tide, the direct dynamical effects of the tidal movements are very slight and belong to the astronomical class, and are in fact merged in the lunar and solar nutations. But if the Earth's surface were divided by meridional barriers, so that the tidal flow would be in the main north and south instead of around the Earth, we might expect a tidal aberration in the latitude of amount not entirely insensible.

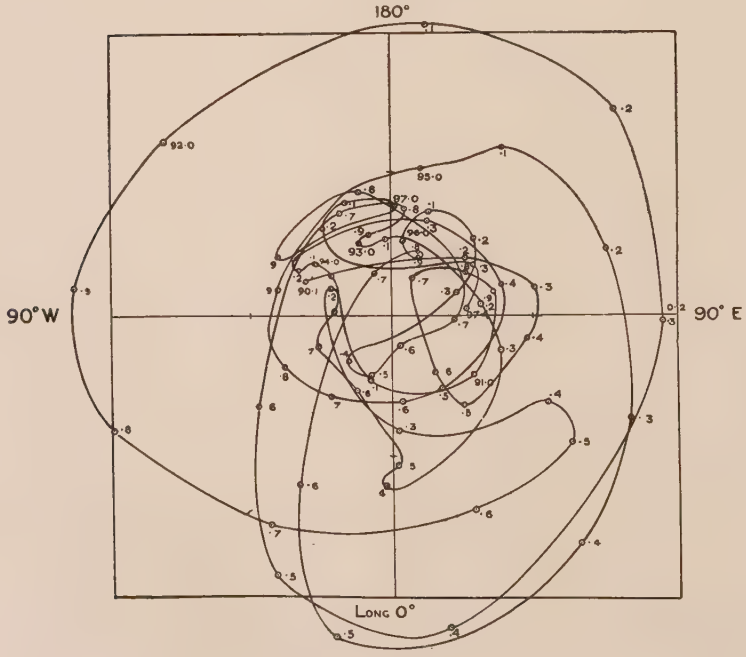
The magnitude of this tidal torque would then compare even with that of the precessional couple of the Moon's attraction on the protuberant parts of the terrestrial spheroid. The latter is $\frac{2}{5}\,\mathrm{M}r^{-3}\,(\mathrm{C}-\mathrm{A})\sin 2\delta$, so that its amplitude is about $\frac{3}{5}(0''{\cdot}1\ g/a)\ 10^{-2}\,\mathrm{C}$ while the earth's angular momentum is $\mathrm{C}\omega$. If this torque were to rotate with the Earth for six hours it would produce an angular displacement of the Pole of amount $0''{\cdot}001\ g.6h/\omega a$, where $\omega^2 a/g = \frac{1}{289}$, that is, of amount $0''{\cdot}001 \times 289.\frac{1}{2}\pi$ or $0''{\cdot}5$; in contrast with the actual lunar fortnightly nutation of amplitude one or two seconds. It has just been seen that a partial tidal overflow from the poles covering $(4000)^2$ square miles to the depth of a foot, and carried on with the Earth's rotation, could in an extreme case account for a displacement of the Pole as much as $2''$: and the antipodal high waters reinforce each other. It is true that if the Earth were covered symmetrically with water, the Sun and Moon travelling in the equator would produce no effect. But the obliquity of their paths and the irregular distribution of the oceans must lead indirectly to nutations of the pole of the short periods of the various tidal components, which are not insensible in the present connection. Thus, for example, a forced nutation of this kind might introduce discrepancies into observations around a parallel of latitude, of character in part systematic owing to the progressive semi-diurnal change of phase, which would be eliminated in smoothing out the observations for each observing station of the International chain of longitude.

Possibilities in this direction are perhaps worth bearing in mind.

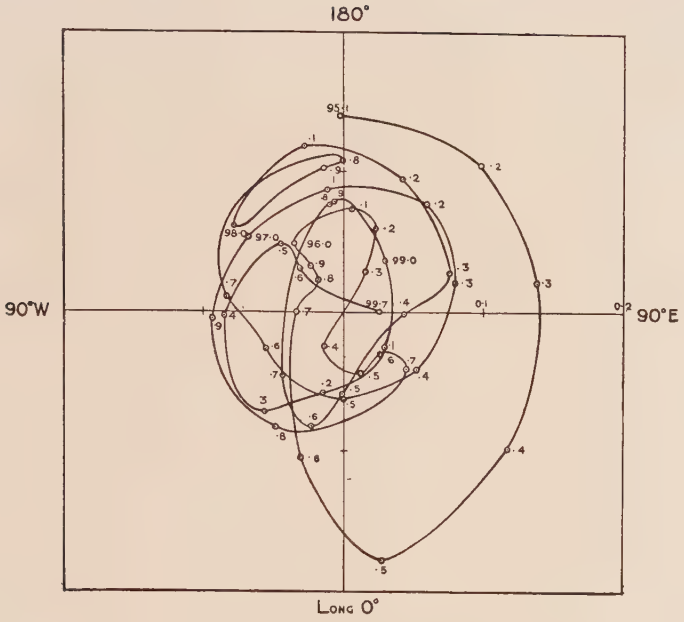
oscillation in latitude of its own period, which would be superposed on the free oscillation of a nearly equal period, thus producing an alternation in its amplitude.



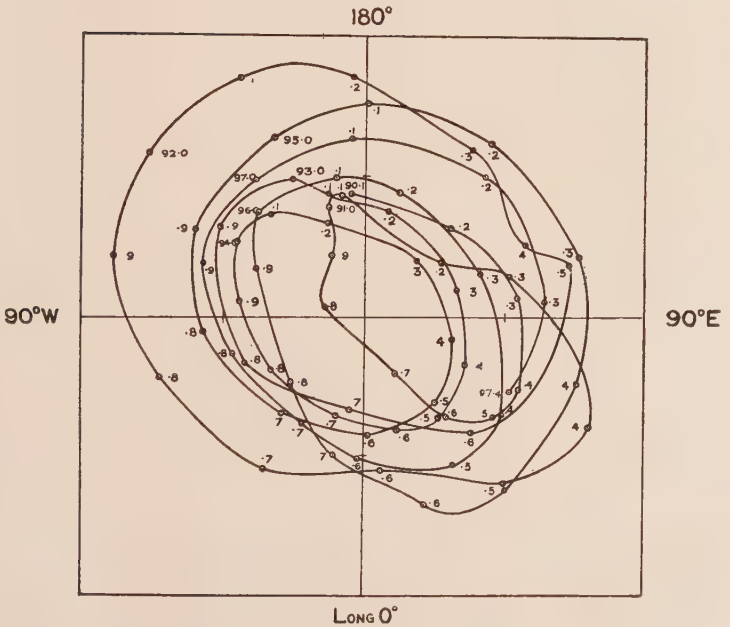
1. ALBRECHT, 1900-1905.



2. ALBRECHT, 1890-1897.



3. CHANDLER, 1895-1899.



4. CHANDLER, 1890-1897.

For example, in addition to the difference of phase at different stations mentioned above, if the same stars were observed at all the stations, a solar tidal nutation might partially simulate a change of latitude with a yearly period, common to all the stations, such as appears in the reduced observations. The deviation of the vertical by the attraction of the tidal water is in all cases very small compared with this deviation of the Pole (*cf.* Thomson and Tait's *Natural Philosophy*, ed. 2), and is in fact here negligible.

The influence of a nutational torque of fairly short period could be laid off by aid of the diagram referred to rotating axes; but even in this case it can be effected rather more easily in the usual astronomical manner, on account of the relatively slow change of its direction in space.

But whether these tidal influences are sensible or not, assuming 428 days to be the period of free precession, we can transform the curve of wandering of the Pole as above to axes of co-ordinates rotating with this period: and the hodograph of this new curve, when referred back again to axes connected with the Earth, will represent the distribution in direction and time of the torque arising from displacement of terrestrial material, which is continually modifying the motion of the Pole. From the point of view of geophysics, this curve would appear to be worth setting out, and might be expected to show seasonal recurrences.

The annexed diagram (1) gives, from the path of the Pole since 1900, as officially published by Professor Albrecht in the *Ast. Nachr.*,* the torque which must have been in action to cause that motion. It will be observed that the expected annual periodicity does not appear, but that the direction of the axis of the torque points preponderantly towards the side of the Pacific Ocean, an extraneous feature which we shall show how to eliminate later (p. 31).

Then follow, but based on more imperfect data, the torque-diagram (2) derived from the Albrecht diagram of Polar wandering for the period 1890–97. The other two are derived from the Chandler diagrams for the periods 1895–99 and 1890–97. It will be noticed that for the first one and a half years of the overlapping period 1895–97 the torque-diagram is much the same in both, but not for the later part. The appearance of all these diagrams is very different from that of the first, *e.g.* the one-sided bias in the torque does not appear.

The unit (0·1) marked on the axes of the diagrams represents the torque that would shift the Pole at the rate of 0″·05 in one-tenth of a year; the dates are marked along the curves in decimals of a year.

The analysis of Chandler made the motion of the Pole consist of his circular precession of 428 days period, with an additional elliptic

* The most convenient way to use these diagrams is to place them on a terrestrial globe, with the origin at the Pole. The Albrecht and Chandler path-diagrams from which these force-diagrams have been constructed may be found in *Bericht über den Stand . . . der Breitvariation*, and *Ast. Journal*, vol. xix., respectively.

motion about the centre, and of yearly period, superposed on it. The component torques that would originate such an addition to a free precession of 428 days, being proportional to $\dot{\omega}_1 - n\omega_2$ and $\dot{\omega}_2 + n\omega_1$, would have an annual elliptic periodicity, in rough agreement with this diagram. For, referring to the axis of the ellipse, this theoretical motion would be

$$\omega_1 = \mathrm{A}\cos(nt+\alpha) + a\cos pt$$
$$\omega_2 = \mathrm{A}\sin(nt+\alpha) + b\sin pt,$$

so that

$$\dot{\omega}_1 - n\omega_2 = -(ap+bn)\sin pt$$
$$\dot{\omega}_2 + n\omega_1 = (an+bp)\cos pt,$$

in which the amplitude A of the free precession is not involved.

We have now to inquire into the kind of information that these diagrams can convey. On our plan of analysis, on the basis of a definite elastic Earth on which additional matter can be displaced, the forcive necessary to supply new angular momentum to material that has come into a position of greater diurnal velocity has to be supplied by this earth, while it has also to sustain the centrifugal force of this material. The other ways in which the mobile material reacts on the motion of the Earth on which it is superposed are negligible in comparison with these two.*

For a mass m in co-latitude θ the centrifugal torque is $m\Omega^2 r^2 \sin\theta\cos\theta$ around an axis at right angles to the meridian of m. The aggregate torque corresponding to its angular momentum in its present position is $m\Omega r^2 \sin\theta\cos\theta$, as regards the equatorial component which is in the meridian of m. The former operates as a whole as a forcive, but only the time-gradient of the latter thus acts, which is $2m\Omega r v\cos 2\theta$, where v is the velocity of m along the meridian. The former preponderates, in the ratio $\frac{1}{2}\Omega r$ to v, therefore usually very much so, as $\frac{1}{2}\Omega r$ is of the order of 300 miles per hour. On the other hand, the centrifugal force of a new steady local load merely makes the steady precession occur about a new axis of inertia, thus is not progressive or cumulative. This is readily verified by reversing the graphical procedure; a steady torque, represented by the end of a radius vector, becomes represented by an arc of a circle when referred to axes rotating with the free precessional velocity, and this corresponds to a velocity of circular precession of the Pole while it lasts.

This double mode of action of transported material, through centrifugal force and through change of momentum of diurnal motion, renders interpretation of the torque-diagram to some extent indefinite.

But, neglecting the effect of change of intrinsic angular momentum, which may be as much as one-tenth of the whole, the torque will be due to the centrifugal force of the distribution

* This is readily seen by the procedure of the note, p. 24, if we introduce the exact formula $h_1 = \mathrm{A}'\omega_1 - \mathrm{E}\omega_3 - \mathrm{F}\omega_2 + \mathrm{I}\omega_1$, where for an additional mass m at xyz, $\mathrm{F} = mxy$, $\mathrm{E} = mxz$; we are in fact merely neglecting $\Omega\omega_1$ and $\Omega\omega_2$ compared with Ω^2.

of the mobile load at each instant, and will thus indicate the general features of that distribution; while the rate of change of the torque, *i.e.* the velocity in this torque-diagram, will give an indication of the movement of the load. The radius vector O P of the torque-diagram at any instant will imply a proportional accumulation of materials in middle latitudes on the meridian at right angles to O P, so that antipodal accumulations reinforce, but adjacent ones, north and south of the equator, counteract each other. The marked tendency of the torque-diagram for the period 1900–05 towards the side of the Pacific Ocean might thus be due to simultaneous accumulation of load not on the side of the Pacific, but in the neighbourhood of the perpendicular meridian. There is, however, a possible alternative to be kept in view, as follows.

It has been suggested to us by Professor Turner that the position of the origin to which the curve of wandering of the Pole of rotation is referred, is subject to considerable uncertainty. The observations and their reduction do not, however, seem to be at fault specially in this direction; unless the unexplained constant (Kimura) term may be taken to indicate a radius of uncertainty due to seasonal instrumental changes. This term, which recent discussion has confined to a smaller amplitude and to an annual period, was referred roughly to a displacement (mainly N and S) of the Earth's centre of gravity: we have verified above, however, that no likely meridional transfer due to seasonal change of temperature could produce an effect so great.

There does not, in fact, seem to be any ground, apart from mere uncertainties, for taking the origin to which the wanderings of the Pole of rotation are referred to be other than a fixed point on the Earth. But, on the other hand, this fixed point may not be the Pole of inertia of the solid Earth. We can, however, make it so, by separating from the solid Earth a thin superficial sheet, and counting this with the mobile material. The centrifugal forcive due to this sheet will then constitute a torque invariable with respect to the Earth: and we have merely to subtract this from the torque-diagram referred to the Earth in order to obtain the torque due to the loose material alone. This subtraction of a constant vector term amounts simply to a change of origin. Thus on the final torque diagram the origin is uncertain, and would naturally be placed in as central a position as possible.

The process here carried out graphically may be compared with the procedure by successive steps as employed by Newcomb, in which free precession occurs for an infinitesimal time δt round an axis of inertia O supposed fixed in the Earth, then O is moved on to O_1 as the result of the change of the mobile load during that time, then free precession takes place round O_1 for a time δt_1, then O_1 is moved on to O_2, etc. Our method has virtually amounted to the elimination of the free precessional motion, with its constant angular velocity, thus leaving the causes which displace the pole of inertia on the Earth's surface open to inspection—the constrained

shift of the axis of rotation in space being neglected for the nearly spherical Earth, as *infra*.

How far the results may throw light on their causes depends largely on a comparison of the diagrams with the displacements of matter on the Earth's surface that are known to meteorology and oceanography. It may be that not much certain information may yet be derivable; but, considering the long time that observations of the wandering of the Pole have been accumulating, it can hardly be said that it is too soon to prepare for their preliminary discussion from the geophysical point of view.

The mode of reduction on which this paper is founded gives a force diagram which exhibits the torque sustained by the rest of the Earth owing to the displacement in and over it of the movable masses treated as independent bodies. It remains valid, however rapid the free precession may be. In the case of the actual earth the latter is slow, being $C/(C-A)$ sidereal days, in which the difference of moments of inertia has its effective or dynamical value, thus lengthening the free period from 306 to 428 solar days. In this case various features assume simple forms, as appears in the Poinsot representation by a rolling ellipsoid. When this ellipsoid is very nearly spherical, the axis of rotation, drawn from its fixed centre to the point of contact with the plane on which it rolls, is at a small inclination to the invariable direction of resultant momentum (which is normal to that plane) compared with its inclination to the axis of inertia; thus the axis of rotation is practically fixed in direction in space—except as regards the superposed luni-solar precession. When the ellipsoid of inertia is nearly of revolution, as in the case of the Earth, the pole of inertia thus revolves in space with the uniform free precessional velocity, around the fixed direction of the pole of rotation, while at the same time it is undergoing such shifts as the redistribution of material geometrically requires. Our procedure in the above has been to eliminate the uniform precession, and the residue is a graphical representation of the irregular shifts, or rather of the torques which produce them.

The discussion of the question whether in past geological history the pole of rotation has wandered extensively in the Earth seems also capable of being based on simple graphic representation. We shall assume, as before, that the dynamics of the Earth's rotation are based at each instant on a simple kinetic energy T given by

$$2T = A\omega_1^2 + B\omega_2^2 + C\omega_3^2,$$

where A, B, C are effective moments of inertia; it follows that the angular momentum (L, M, N) is given by the formula

$$(L, M, N) = (A\omega_1, B\omega_2, C\omega_3)$$

During the history of the Earth (L, M, N), resultant G, must have remained constant, while T will probably have diminished through

frictional agency. Abstraction is here made of the solar and lunar forced precessions, which compensate the torque of extraneous attractions without affecting the position of the axis of rotation in the Earth. The free motion is represented, after Poinsot, by the angular motion of a momental ellipsoid, say

$$Ax^2 + By^2 + Cz^2 = K$$

The direction of the axis of instantaneous rotation intersects its surface at the point (xyz) such that

$$\frac{x}{\omega_1} = \frac{y}{\omega_2} = \frac{z}{\omega_3}, \text{ therefore} = \left(\frac{K}{2T}\right)^{\frac{1}{2}}$$

The distance p of the tangent plane at this point (xyz) from the centre is given by

$$\frac{1}{p^2} = \frac{A^2}{K^2}x^2 + \frac{B^2}{K^2}y^2 + \frac{C^2}{K^2}z^2$$
$$= \frac{G^2}{2KT}$$

Thus if $K^{-1} = 2T$, we have $p = G^{-1} = \text{constant}$; this tangent plane is then fixed as regards distance from the centre, as well as regards its direction, which is perpendicular to the invariable momentum (L, M, N). Thus it is entirely fixed.

Thus the free precessional motion of the slowly changing Earth is represented by the varying ellipsoid

$$Ax^2 + By^2 + Cz^2 = (2T)^{-1}$$

rolling with centre fixed, so as to keep in contact with this fixed plane whose distance from the centre is G^{-1}.

If the kinetic energy keeps constant, this is simply Poinsot's representation. The axis of rotation will circulate in the body around the axis of greatest moment of inertia, and in space around the axis of resultant angular momentum. The amplitude of this free precession will be kept small by internal friction, so that the axis of rotation will always be near the principal axis, and can never wander further from its original position than the latter does. It will require a good deal of change of distribution of mass to move this principal axis very far: to move it into the equator the radius of the Earth must shrink to the order of 40 miles along the equator near the new Poles, and expand to about an equal extent near the original Poles.

If the Earth is shrinking uniformly, the moments of inertia vary as l^2, where l represents linear dimensions; thus the angular velocities vary as l^{-2} and the kinetic energy varies as l^{-2}. Thus the dimensions of this rolling ellipsoid remain unaltered. The distance of the plane on which it rolls also keeps fixed. Hence uniform shrinkage without frictional loss of energy would not affect the amplitude of the free precession, or cause the Poles to migrate.

Diminution of the energy of rotation through internal friction, without change of (A, B, C), would increase the dimensions of the rolling ellipsoid (in the proportion of $T^{-\frac{1}{2}}$), and so would make strongly for stability of the axis of rotation, as above remarked. Such increase can only proceed to a limited extent, determined by the ellipsoid just failing to intersect, and so touching the fixed plane at the extremity of its axis.

A discussion of the geological problem of displacement of the polar axis in the Earth must take account of considerations such as these.

The Systematic Motions of the Stars.

By A. S Eddington, B.A., M.Sc.

1. *Introduction*

Of late years astronomers who have investigated the proper motions of stars have been mainly interested in determining the direction of the solar motion. It is usual to assume that, if we consider a sufficient number of stars, their true motions will be at random; on this assumption the direction of the sun's motion has been calculated.

Professor J. C. Kapteyn * has examined the Bradley proper motions to find out whether this assumption is approximately true. He concludes that it is incorrect. Relative to the sun, he finds two "favoured" directions of motion instead of one. Kapteyn suggested that there are two systems or "drifts" of stars; these two drifts are in motion relative to one another. If the whole universe forms one system (or one chaos) we can speak of its motion relative to the sun; but it is more natural, though perhaps misleading, to speak of the sun's motion relative to it. But if there are two systems, we may as well drop the idea of the solar motion altogether, and speak of the motions of the two drifts relative to the sun.

In this paper I have attempted to subject Kapteyn's theory to a quantitative test by examining the Greenwich-Groombridge proper motions. I was led to undertake this by the following considerations:—

(1) The importance and revolutionary character of Kapteyn's discovery render independent confirmation very desirable.

(2) The Greenwich-Groombridge proper motions, recently determined by Dyson and Thackeray, afford new material for this purpose. I had the advantage of access to Dyson and Thackeray's calculations used in their determination of the so-called solar apex.

(3) The Bradley stars are all bright stars. The Groombridge catalogue includes a large proportion of stars between the seventh and ninth magnitudes; it is desirable to find out whether these fall into Kapteyn's two drifts.

* *Brit. Assoc. Report*, 1905.

(4) The Groombridge stars number about 4500 and are all within 52° of the north pole. The 2500 Bradley stars cover most of the sky. For some purposes there is an advantage in comparing the phenomena in different parts of the sky, but for quantitative tests of the theory I have found density of distribution essential; this the Groombridge catalogue affords.

I have also examined in the same manner the Greenwich-Carrington proper motions.

When more determinations of line-of-sight velocities are published, perhaps a complete demonstration of the two-drift theory will be possible. What is here attempted is to show that there exist anomalies of a remarkable and systematic character in the proper motions (or what are believed to be the proper motions) of the stars; and that these anomalies are, so far as our evidence goes, entirely explained if we suppose the visible universe to consist in the main of two streams of stars crossing through one another.

The test of a hypothesis is its power of predicting facts. It is easy to pick out half-a-dozen stars which, our theory predicts, will have an average velocity towards the sun, and another half-dozen from the same part of the sky which will have a velocity away from the sun.

Until some such simple test is applied, it would be wrong to regard the theory as established. On the other hand, it must not be forgotten that, should the two-drift theory fail when the crucial test is applied, some other explanation will have to be found for the remarkable phenomena to which Kapteyn has called attention.

2. *Mathematical Theory.*

I define a "drift of stars" to be a system of stars whose velocities relative to some system of axes are quite haphazard. The velocity of the drift is the velocity of the aforesaid system of axes. The "peculiar" velocity of a star is its velocity relative to that system. For example, the ordinary hypothesis on which discussions of the solar motion are based regards the universe as forming one such drift. It is sometimes further assumed that this drift is at absolute rest; but, of course, referred to the sun, it is in relative motion.

I shall accept, as the best mathematical equivalent of "haphazard," a distribution of velocities according to Maxwell's law. Of course there is no analogy between the causes which determine the velocities of the molecules of a gas and those of the stars. In the former case it is the collisions which make the molecules obey this law. But Jeans* has shown that of all distributions *having the same energy*, that according to Maxwell's law is the most probable; and as the number of bodies increases, the probability of a Maxwellian distribution approaches certainty.

* *The Dynamical Theory of Gases*, § 56. We take as our *basis of probability* that all values of a velocity component are equally probable.

For our purposes it will be sufficient to accept the Maxwellian distribution as a *standard*; and it will be the object of the comparison of observation and theory to discover any marked deviations from this standard distribution of stellar velocities.

Consider a small region of the sky which can be considered approximately plane, and suppose the stars in it belong to one drift. We shall consider their *linear* velocities, but ignore motion in the line of sight.

Let the drift velocity be V along Ox, and let the number of stars having component peculiar velocities between (u, v) and $(u+du, v+dv)$ be in accordance with Maxwell's law

$$Ae^{-h^2(u^2+v^2)}du\,dv$$

I shall call h the drift-constant; it is connected with the mean peculiar speed Ω of the stars of the drift by the relation

$$\Omega = \sqrt{\frac{4}{\pi}} \cdot \frac{1}{h}.$$

If r be the resultant velocity, θ its inclination to Ox, we shall have

$$u^2+v^2 = r^2 + \mathrm{V}^2 - 2\mathrm{V}r\cos\theta$$
$$du\,dv = r\,dr\,d\theta$$

Hence the number of stars having proper motions in directions inclined to Ox between θ and $\theta+d\theta$

$$= \mathrm{A}d\theta \int_0^\infty e^{-h^2(r^2+\mathrm{V}^2-2\mathrm{V}r\cos\theta)} r\,dr$$

or putting $z = r - \mathrm{V}\cos\theta$,

$$= \mathrm{A}d\theta e^{-h^2\mathrm{V}^2\sin^2\theta} \int_{-\mathrm{V}\cos\theta}^\infty e^{-h^2z^2}(z+\mathrm{V}\cos\theta)dz$$

now put $hz = x$ and $h\mathrm{V}\cos\theta = \tau$ the number becomes

$$= \frac{\mathrm{A}}{h^2} e^{-h^2\mathrm{V}^2} d\theta \left\{ e^{\tau^2} \int_{-\tau}^\infty e^{-x^2}(x+\tau)dx \right\}$$

$$= \mathrm{B}d\theta \left\{ \tfrac{1}{2} + \tau e^{\tau^2} \int_{-\tau}^\infty e^{-x^2}dx \right\}$$

where B is independent of θ.

The above function of τ is very important in what follows. Table I. gives the values of $\log f(\tau)$, where

$$f(\tau) = \frac{2}{\sqrt{\pi}} \left\{ \tfrac{1}{2} + \tau e^{\tau^2} \int_{-\tau}^\infty e^{-x^2}dx \right\}$$

Table I.

τ.	$\log f(\tau)$.	τ.	$\log f(\tau)$.	τ.	$\log f(\tau)$.	τ.	$\log f(\tau)$.
−1·3	$\bar{2}$·997	−·4	$\bar{1}$·471	·5	0·188	1·3	1·152
−1·2	$\bar{1}$·041	−·3	·536	·6	·288	1·4	·300
−1·1	·087	−·2	·605	·7	·395	1·5	·455
−1·0	·135	−·1	·676	·8	·506	1·6	·618
−·9	·185	0	·751	·9	·623	1·7	·787
−·8	·238	·1	·830	1·0	·746	1·8	·964
−·7	·292	·2	·913	1·1	·875	1·9	2·148
−·6	·349	·3	0·000	1·2	1·010	2·0	·339
−·5	·409	·4	·091				

Consider the curve obtained by drawing the radius vector in any direction proportional to the number of stars having proper motions in that direction.

The polar equation of the curve will of course be $r \propto f(h\mathrm{V}\cos\theta)$,

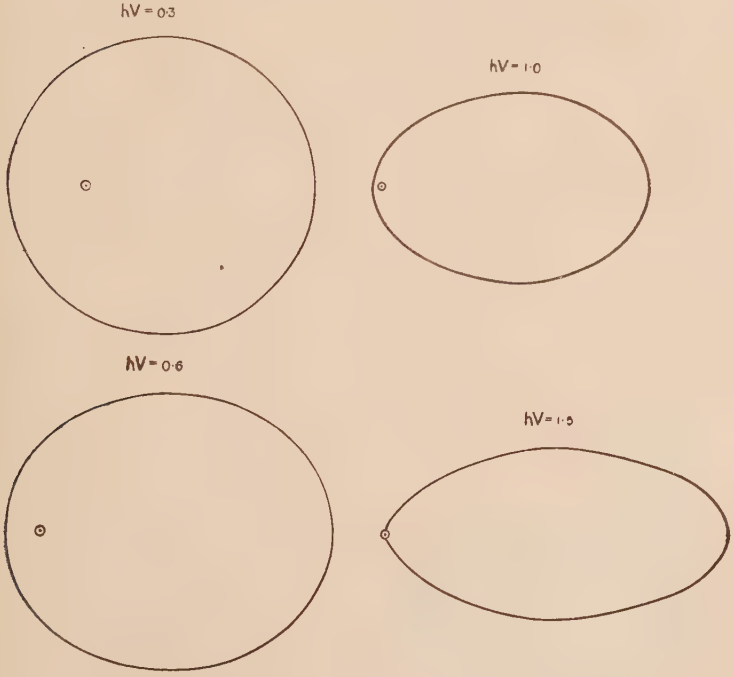


Fig. 1.

or more generally, if the drift is in the direction θ_0 instead of being along the initial line,

$$r \propto f(h\mathrm{V}\cos\overline{\theta - \theta_0})$$

The shape of the curve depends on the parameter $h\mathrm{V}$; in other words, it depends on the ratio of the drift velocity to the mean peculiar velocity. It does not in any way depend on the distances of the stars.

Fig. 1 shows the curves drawn for four different values of $h\mathrm{V}$. It will be noticed how rapidly the shape of the curve changes for small variations in $h\mathrm{V}$.

3. *Treatment of the Observed Proper Motions.*

The region covered by the Groombridge catalogue comprises that part of the sky within 52° of the north pole. I divided it into seven regions as follows:—

Region A	N.P.D. 0°–20°	
,, B	N.P.D. 20°–52°	R.A. 22^h– 2^h
,, C	,,	,, 2^h– 6^h
,, D	,,	,, 6^h–10^h
,, E	,,	,, 10^h–14^h
,, F	,,	,, 14^h–18^h
,, G	,,	,, 18^h–22^h

In making this division it was necessary to steer between two evils. If too large a region is chosen, the proper motions will not follow the same law at all parts of it, and they cannot fairly be treated as all in one plane. But the regions must be large enough to contain a sufficient number of stars; it is difficult to deal mathematically with a region containing less than 400 stars.

In discussing one of these regions, a table was first made showing how many stars are moving in each direction. The angle θ which indicates the direction was measured in a counter-clockwise direction (as seen in the sky) from the direction of increasing R.A. at the centre of the region.* Thus $\theta = 90°$ would mean towards the equator and parallel to the central meridian.

Fig. 2 shows curves drawn to represent these tables. As in the theoretical curves of fig. 1, the radius vector in any direction is proportional to the number of stars whose proper motions are in that direction.

Let us consider, for example, the curve B (corresponding to region B), and compare it with the theoretical single-drift curves of fig. 1. Perhaps the most remarkable feature is the extraordinary minimum between *oa* and *ob*. The number of stars having directions of motion within this right angle is found on reference to the table to be only 63 out of 862. But the curve differs

* In the case of region A, the direction $\theta = 0°$ is along the meridian 6^h.

completely from the theoretical curves in that the maximum is not by any means opposite to this, but is along *oc*; in fact, there is a sort of secondary minimum along *od*, where we should have expected the maximum to be.

It is no exaggeration to say that the hypothesis that the stars

FIG. 2.

form a single drift does not represent with the slightest approximation the observed distribution of proper motions in the case of region B at any rate. No matter what orientation or parameter of the theoretical curve is taken, at the very least the discordance between theory and observation will amount to 200 stars in one quadrant. Regions C, F, and G in like manner contradict the hypothesis of one drift, whilst there are serious discordances in the

cases of D and E. In region A we have two maxima of the radius vector nearly opposite one another, so that the disagreement with theory is in some respects more remarkable still. Kapteyn in his investigation found this same disagreement, and pointed out the explanation—that there are really two drifts of stars.

I have accordingly analysed each region separately to see if it can be accounted for on the assumption that each curve is compounded of two simple drift curves. For instance, in fig. 3, X and Y are theoretical curves for single drifts; X represents a drift con-

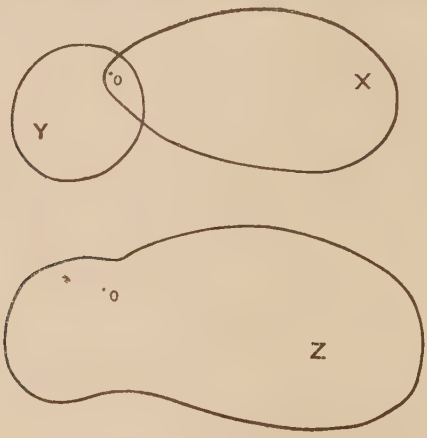


Fig. 3.

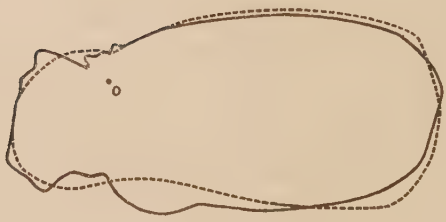


Fig. 4.

taining 418 stars, and for which $hV = 1{\cdot}55$; Y a drift containing 430 stars, and for which $hV = {\cdot}45$. The directions of motion of the two drifts are inclined at an angle of 120°. The compound curve Z is drawn with its radius vector in any direction equal to the sum of the radii vectores of X and Y in that direction. It therefore corresponds to a mixture of the stars of the two drifts above mentioned. The curve Z is the best approximation to fig. 2B which I am able to obtain by combining two simple curves. Fig. 4 shows a comparison of the two. There is a small difference in the two curves between the radii $\theta = 50°$ and 80°, amounting to an

excess of 24 stars altogether (*i.e.* the observed curve has 114 stars, where the theoretical curve has 90 stars). Elsewhere the agreement is practically perfect. For this and the other regions, the agreement is better seen by referring to the tables which follow, and especially to figs. 5 and 6.

In Tables II. to VII., corresponding to various values of θ, the fifth column shows the actual number of stars having proper motions in that direction *—the radius vector of the diagrams. The fourth column shows the theoretical number given by my analysis, and the second and third columns show how the two drifts contribute to this theoretical number. The comparison of the observed and theoretical numbers is shown in figs. 5 and 6 for four regions. The number of stars and the corresponding direction of motion θ are plotted as ordinate and abscissa instead of (as in the previous diagrams) as polar co-ordinates.

It must not be supposed that I claim that two drifts represent completely the phenomena of the distribution of proper motions, and that the discordances are necessarily either errors of observation or the purely accidental departures which the theory of probability allows. Local irregularities must be expected; perhaps a third smaller drift exists throughout the sky, or one of the two principal drifts may prove to be compound. What is claimed for the two-drift hypothesis is that it is a good first approximation, whereas the one-drift hypothesis is no approximation at all.

Each region has been analysed without reference to the others. The six disposable constants of each theoretical curve have been determined by trial and error, so as to make it coincide as nearly as possible with the observed curve. Of course, if the same two drifts persist from region to region, there must be relations between the constants of the drifts in the different regions. The fulfilment of these relations is a good test of the theory.

* More precisely—one-third the number of stars having proper motions in directions between $\theta - 15°$ and $\theta + 15°$. The other three columns have been "smoothed" in the same way.

TABLE II.—REGION A.

Direction θ.	Calculated.			Observed.	Difference observed—calculated.
	Drift I.	Drift II.	Total.		
5°	42	5	47	49	+2
15°	35	5	40	39	−1
25°	26	5	31	27	−4
35°	16	5	21	20	−1
45°	9	6	15	18	+3
55°	5	7	12	15	+3
65°	2	8	10	14	+4
75°	1	8	9	12	+3
85°	1	9	10	12	+2
95°	1	10	11	11	0
105°	0	11	11	10	−1
115°	0	12	12	10	−2
125°	0	12	12	9	−3
135°	0	13	13	13	0
145°	0	13	13	11	−2
155°	0	13	13	14	+1
165°	0	13	13	12	−1
175°	0	13	13	14	+1
185°	0	12	12	12	0
195°	0	12	12	13	+1
205°	0	11	11	11	0
215°	0	10	10	10	0
225°	0	9	9	7	−2
235°	0	8	8	7	−1
245°	0	8	8	9	+1
255°	0	7	7	9	+2
265°	1	6	7	7	0
275°	1	5	6	7	+1
285°	1	5	6	9	+3
295°	2	5	7	11	+4
305°	5	5	10	12	+2
315°	9	5	14	15	+1
325°	16	4	20	21	+1
335°	26	4	30	29	−1
345°	35	4	39	39	0
355°	42	5	47	47	0
Totals	276	293	569	585	

TABLE III.—REGION B.

Direction θ.	Calculated.			Observed.	Difference observed—calculated.
	Drift I.	Drift II.	Total.		
5°	59	6	65	67	+2
15°	59	7	66	65	−1
25°	51	8	59	54	−5
35°	38	9	47	47	0
45°	25	10	35	37	+2
55°	15	12	27	31	+4
65°	8	14	22	30	+8
75°	5	16	21	28	+7
85°	2	18	20	25	+5
95°	1	20	21	19	−2
105°	1	22	23	20	−3
115°	1	23	24	20	−4
125°	1	24	25	26	+1
135°	1	24	25	25	0
145°	0	23	23	26	+3
155°	0	22	22	22	0
165°	0	20	20	20	0
175°	0	18	18	18	0
185°	0	16	16	18	+2
195°	0	14	14	15	+1
205°	0	12	12	16	+4
215°	0	10	10	10	0
225°	0	9	9	5	−4
235°	0	8	8	7	−1
245°	1	7	8	6	−2
255°	1	6	7	6	−1
265°	1	6	7	7	0
275°	1	5	6	6	0
285°	1	5	6	8	+2
295°	2	5	7	8	+1
305°	5	5	10	10	0
315°	8	5	13	13	0
325°	15	5	20	18	−2
335°	25	5	30	27	−3
345°	38	5	43	42	−1
355°	51	6	57	60	+3
Totals .	416	430	846	862	

TABLE IV.—REGION C.

Direction θ.	Calculated.			Observed.	Difference observed—calculated.
	Drift I.	Drift II.	Total.		
5°	5	6	11	11	0
15°	9	7	16	15	−1
25°	16	7	23	24	+1
35°	24	8	32	34	+2
45°	32	9	41	42	+1
55°	35	9	44	43	−1
65°	32	10	42	38	−4
75°	24	11	35	35	0
85°	16	12	28	27	−1
95°	9	13	22	23	+1
105°	5	13	18	16	−2
115°	2	14	16	14	−2
125°	1	14	15	12	−3
135°	1	14	15	13	−2
145°	1	13	14	14	0
155°	0	13	13	14	+1
165°	0	12	12	12	0
175°	0	11	11	11	0
185°	0	10	10	10	0
195°	0	9	9	10	+1
205°	0	9	9	9	0
215°	0	8	8	8	0
225°	0	7	7	7	0
235°	0	7	7	6	−1
245°	0	6	6	6	0
255°	0	6	6	4	−2
265°	0	5	5	4	−1
275°	0	5	5	4	−1
285°	0	5	5	5	0
295°	0	5	5	5	0
305°	0	5	5	6	+1
315°	0	5	5	7	+2
325°	1	5	6	7	+1
335°	1	5	6	6	0
345°	1	5	6	6	0
355°	2	6	8	8	0
Totals	217	309	526	516	

TABLE V.—REGION D.

Direction θ.	Calculated.			Observed.	Difference observed—calculated.
	Drift I.	Drift II.	Total.		
5°	0	5	5	4	−1
15°	0	5	5	5	0
25°	1	5	6	5	−1
35°	1	5	6	7	−1
45°	1	5	6	8	+2
55°	2	5	7	9	+2
65°	4	5	9	12	+3
75°	6	6	12	12	0
85°	10	6	16	16	0
95°	16	6	22	20	−2
105°	22	6	28	30	+2
115°	27	7	34	35	+1
125°	29	7	36	37	+1
135°	27	7	34	30	−4
145°	22	7	29	27	−2
155°	16	8	24	22	−2
165°	10	8	18	19	+1
175°	6	8	14	16	+2
185°	4	8	12	15	+3
195°	2	8	10	15	+5
205°	1	8	9	10	+1
215°	1	8	9	8	−1
225°	1	7	8	6	−2
235°	0	7	7	7	0
245°	0	7	7	8	+1
255°	0	7	7	8	+1
265°	0	6	6	6	0
275°	0	6	6	5	−1
285°	0	6	6	5	−1
295°	0	6	6	5	−1
305°	0	5	5	5	0
315°	0	5	5	5	0
325°	0	5	5	5	0
335°	0	5	5	5	0
345°	0	5	5	6	+1
355°	0	5	5	5	0
Totals .	209	225	434	443	

TABLE VI.—REGION F.

Direction θ.	Calculated.			Observed.	Difference observed—calculated.
	Drift I.	Drift II.	Total.		
5°	0	6	6	4	−2
15°	0	7	7	5	−2
25°	0	8	8	6	−2
35°	0	10	10	9	−1
45°	0	11	11	10	−1
55°	0	12	12	14	+2
65°	0	12	12	14	+2
75°	0	13	13	14	+1
85°	0	13	13	13	0
95°	0	12	12	12	0
105°	1	12	13	10	−3
115°	1	11	12	11	−1
125°	1	10	11	10	−1
135°	1	8	9	10	+1
145°	2	7	9	7	−2
155°	3	6	9	9	0
165°	5	6	11	9	−2
175°	7	5	12	14	+2
185°	11	4	15	14	−1
195°	15	4	19	16	−3
205°	19	3	22	21	−1
215°	23	3	26	27	+1
225°	24	3	27	29	+2
235°	23	3	26	26	0
245°	19	3	22	19	−3
255°	15	3	18	17	−1
265°	11	3	14	12	−2
275°	7	3	10	11	+1
285°	5	3	8	11	+3
295°	3	3	6	8	+2
305°	2	3	5	7	+2
315°	1	3	4	6	+2
325°	1	4	5	6	+1
335°	1	4	5	5	0
345°	1	5	6	5	−1
355°	0	6	6	4	−2
Totals .	202	232	434	425	

TABLE VII.—REGION G.

Direction θ.	Calculated.			Observed.	Difference observed—calculated.
	Drift I.	Drift II.	Total.		
5°	26	9	35	39	+4
15°	22	11	33	39	+6
25°	17	12	29	34	+5
35	13	14	27	28	+1
45°	11	16	27	27	0
55°	8	18	26	28	+2
65°	7	21	28	30	+2
75°	5	24	29	29	0
85°	5	26	31	31	0
95°	4	29	33	34	+1
105°	3	30	33	33	0
115°	3	31	34	32	−2
125°	3	31	34	34	0
135°	3	30	33	35	+2
145°	3	29	32	33	+1
155°	3	26	29	27	−2
165°	3	24	27	28	+1
175°	3	21	24	27	+3
185°	3	18	21	24	+3
195°	4	16	20	22	+2
205°	5	14	19	20	+1
215°	5	12	17	20	+3
225°	7	11	18	17	−1
235°	8	9	17	18	+1
245°	11	8	19	19	0
255°	13	8	21	20	−1
265°	17	7	24	25	+1
275°	22	7	29	29	0
285°	26	6	32	32	−0
295°	32	6	38	34	−4
305°	36	6	42	38	−4
315°	40	6	46	46	0
325°	41	7	48	48	0
335°	40	7	47	46	−1
345°	36	8	44	40	−4
355°	32	8	40	37	−3
Totals .	520	566	1086	1103	

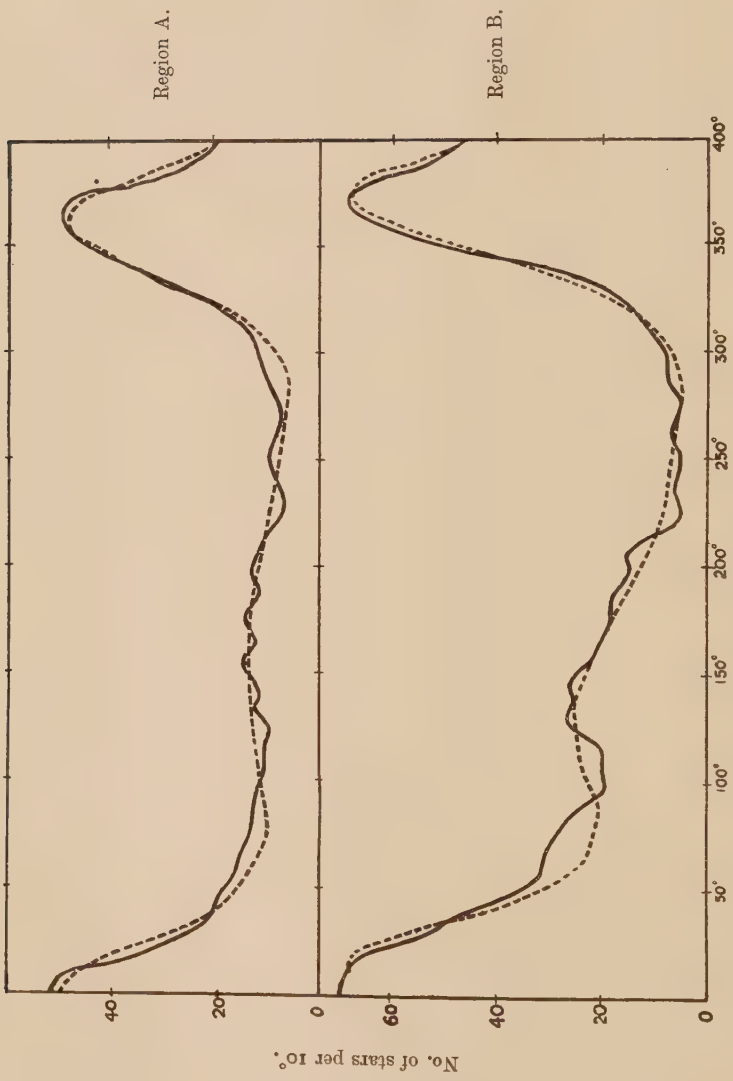


FIG. 5.—Comparison of observation and theory. Continuous line *observed*, dotted line *theory*.

FIG. 6.—Comparison of observation and theory.

4. *Results of the Analysis.*

The following are the constants of the drifts in the several regions. They are the values on which the columns headed "calculated" in the preceding tables are based.

TABLE VIII.

		Drift I.	Drift II.
Region A	Number of stars	277	293
	kV	1·65	·30
	θ_0 (direction of drift)	0°	155°
Region B	Number of stars	418	430
	kV	1·55	·45
	θ_0	10°	130°
Region C	Number of stars	217	309
	kV	1·65	·30
	θ_0	55°	125°
Region D	Number of stars	209	225
	kV	1·40	·15
	θ_0	125°	185°
Region F	Number of stars	202	232
	kV	1·20	·45
	θ_0	225°	80°
Region G	Number of stars	520	566
	kV	·75	·45
	θ_0	325°	120°

Region E contained too few stars to analyse satisfactorily.

Using a globe, I entered the directions of the drifts as great circles on it. The great circles were drawn from the centres of regions (which in the case of regions B C D G were assumed to be at N.P.D. 40°*) in the directions indicated by θ_0 in the above results. Supplementing the drawing by calculation, I found that for drift I. the great circles (produced backwards) all passed close to the point R.A. 18^h, Decl. $+18°$. The accuracy was rather surprising: five circles passed within 2° of the point; that for region G was about 5° out. The stars of drift I. have clearly a common velocity relative to the sun away from this point. I shall call the point Q_1. In spite of the close agreement of the great circles, they intersect at so acute an angle, all coming from

* This was verified by actual calculation of the centre of mean position of the stars in several regions. For F the centre was taken to be N.P.D. 36°; it was discovered that some stars between 0° and 20° N.P.D. had accidentally been included in the region. As the number of stars in region F would have been too few to analyse if they were excluded, the results for this augmented region have been retained.

a limited region of the sky, that Q_1 is not very accurately determined, especially as regards declination.

Theoretically, hV should be proportional to the sine of the angular distance of the area from Q_1, for V is the component velocity of the drift *across the line of sight*. The following table shows a comparison between observation and theory. The decrease in hV as we approach Q_1 is indicated, but the agreement is only rough. In particular, region G shows much too small a value.

Region.	Dist. from Q_1.	Sine of Distance.	hV.
A	72°	·95	1·65
B	76°	·97	1·55
C	107°	·96	1·65
D	107°	·96	1·40
F	43°	·68	1·20
G	40°	·64	·75

Examining the results for drift II. in the same way, we must bear in mind that the same degree of accuracy is not to be expected. The drift II. curves are not very elongated, and we cannot tell with precision which way they are pointing. If we take the velocity of the sun relative to drift I. to be 18 kilometres per sec., an error of 0·1 in hV corresponds to an error of 1 kilometre per sec. An error of 5° in the drift I. apex, or an error of 15° in the drift II. apex, corresponds to an error of about 1½ kilometre per sec.

I find the best position for the drift II. apex Q_2 to be

R.A. $7^h\ 30^m$, Decl. $+58°$

the great circles passing fairly near it, except for region G.

The values of hV are in good agreement with theory, as the following table shows:—

Region.	Dist. from Q_2.	Sine of Distance.	hV.
A	32°	·53	·30
B	59°	·86	·45
C	31°	·52	·30
D	9°	·16	·15
F	60°	·87	·45
G	72°	·95	·45

Partly owing to the very large number of stars in region G, and partly owing to its results not agreeing as well as those of the other regions, I made a special investigation, subdividing it into four divisions. Three of the divisions showed the two drifts, and, as far as I could tell, were in good agreement with theory. But the other division (called Gγ), viz., N.P.D. 40°–52°, R.A. 18^h–20^h, containing 336 stars, was evidently quite exceptional. The diagram for it is shown in fig. 7. It will at once be seen that there is no indication here that we have to deal with a combination of two

drifts. There seems to be a third drift in the direction $\theta = 200°$, which is apparently as important as either of the two fundamental drifts. Whether the explanation is that we have here a third local drift, or whether there is some local error in the proper motions, it will at any rate be admitted that Gγ is an abnormal region, and no good can be done by retaining it in our investigation of the nature of the two main drifts which prevail throughout the sky.

I have accordingly analysed the gnomon-shaped region com-

FIG. 7.

pounded of the three remaining divisions. Its centre is at R.A. $20^h\ 30^m$, N.P.D. 36°.

	Drift I.	Drift II.
Number of stars .	328	432
kV . . .	1·0	·45
θ_0 . . .	330°	100°

The direction of drift II. is now in good agreement with the other regions, and so are the values of kV. The direction of drift I. is rather discordant.

5. *The Carrington Proper Motions.*

After the greater part of the preceding work had been completed, another series of proper motions became available for discussion. These are the result of a comparison between Carrington's Catalogue (mean epoch 1855) and the Greenwich Nine year Catalogue for 1900, not yet published. This series includes about 1100 stars down to magnitude 9·5, all included within 9° of the pole. Of course the Groombridge Catalogue overlaps this; and 90 of the brighter stars are common to the two catalogues.

Thackeray has made a comparison of the proper motions of these 90 stars derived from the two catalogues. From this it is found that the mean accidental difference between Groombridge

and Carrington proper motions is 1·1″ per century in either co-ordinate. If the accidental error of either catalogue be assumed to be the same, the mean accidental error of a proper motion will be 0·8″ per century in either co-ordinate.

The result of the analysis of the Carrington proper motions was as follows:—

	Drift I.	Drift II.
Number of stars .	530	620
hV . . .	1·30	·30
θ_0 . . .	5°	140

A comparison with the results for the corresponding region A of Groombridge shows fairly good agreement. The only noteworthy difference is the value 1·30 of hV for drift I. For region A it is 1·65; and although that value is probably rather high, we should expect a value of at least 1·50.

One naturally looks for the explanation of a small value of hV in large accidental error (see § 7). At first sight the above-mentioned comparison of Groombridge and Carrington seemed to forbid this explanation. However, on reflection it was realised that the comparison only referred to bright stars, viz., those which Groombridge observed. Now, the Carrington catalogue contains 364 stars between magnitude 9·0 and 9·5, whilst Groombridge does not include many stars below magnitude 8·5. The Greenwich places of these fainter stars are subject to considerable accidental error, and we can well suppose that the proper motions are not nearly so good as those of the brighter stars, for which the comparison was made.

Moreover, at about magnitude 8·5 magnitude equation begins to be of importance. In any other region magnitude equation would appear as a systematic error, but in a region symmetrical about the pole it is equivalent to an accidental error.

Analysing the Carrington stars brighter than 8·9 magnitude I found

	Drift I.	Drift II.
Number of stars .	383	386
θ_0 . . .	5°	140°
hV . . .	1·40	·35

The increase in hV was very decided.*

Table IX. shows the analysis. The last column gives the number of stars between magnitude 9·0–9·5; it is interesting to note that the two drifts are conspicuous, so that the law seems to extend beyond the ninth magnitude. The maxima are not sharp nor the minima deep—indicating small values of hV, and presumably large accidental error.

* hV is still too small, but we are entitled to expect that if we still further excluded faint stars a higher value of hV would be obtained. Unfortunately, if we remove another half magnitude, too few stars will be left to verify this.

TABLE IX.—CARRINGTON (mag. 0–8·9).

Direction θ.	Calculated.			Observed.	Difference observed—calculated.	Mag. 9·0–9·5 observed.
	Drift I.	Drift II.	Total.			
5°	51	6	57	57	0	17
15°	48	7	55	54	−1	16
25°	39	7	46	48	+2	18
35°	28	8	36	44	+8	17
45°	18	9	27	33	+6	18
55°	11	10	21	28	+7	15
65°	7	11	18	18	0	12
75°	4	13	17	17	0	14
85°	2	14	16	12	−4	12
95°	2	15	17	13	−4	11
105°	1	16	17	17	0	6
115°	1	17	18	18	0	8
125°	1	18	19	21	+2	10
135°	1	19	20	19	−1	9
145°	1	19	20	21	+1	9
155°	1	18	19	19	0	9
165°	1	17	18	17	−1	11
175°	0	16	16	14	−2	9
185°	0	15	15	12	−3	8
195°	0	14	14	12	−2	7
205°	1	13	14	13	−1	8
215°	1	11	12	12	0	8
225°	1	10	11	9	−2	7
235°	1	9	10	9	−1	6
245°	1	8	9	8	−1	4
255°	1	7	8	9	+1	4
265°	1	7	8	8	0	5
275°	2	6	8	9	+1	7
285°	2	6	8	8	0	7
295°	4	6	10	11	+1	8
305°	7	6	13	16	+3	8
315°	11	5	16	20	+4	8
325°	18	5	23	23	0	9
335°	28	6	34	32	−2	12
345°	39	6	45	44	−1	13
355°	48	6	54	55	+1	14
Totals .	383	386	769	780	.	364

6. *Characteristics of the Drifts.*

It will have been noticed that the velocity of the first drift relative to the sun is very much greater than that of the second, the values being about $\frac{1\cdot7}{h}$ and $\frac{\cdot5}{h}$ respectively. For this reason the first drift is very much more prominent than the second in the diagrams. But the result of the analysis is to show that they contain nearly the same number of stars, drift II. containing a slightly greater proportion. Another very unexpected result is that the proportion of drift II. to drift I. stars is very nearly constant in the different regions, as can be seen by inspection of Table VIII. The distribution of the stars is exceedingly irregular, and we should hardly have expected the two drifts to possess the same irregularities. We know, for instance, that stars are much more numerous near the Galactic plane than elsewhere: we might expect this to be a peculiarity due to one drift alone. Now, the Galactic plane passes through region B and part of G, and we see that actually the special abundance of stars there is due to both drifts equally.

We now further inquire whether this constancy of proportion extends to all distances. Do the two drifts form a homogeneous mixture? We have seen that the proportion is the same in all parts of the sky (that we have investigated), but may not the average distance of the drift II. stars be different from that of the drift I.? For instance, may not drift II. be a cluster of stars containing the sun, and moving relatively to drift I., which represents the main background of more distant stars?

Mean Distances of the Stars.—We have deduced the various constants connected with the motions of the two drifts of stars without making use of the *magnitudes* of the proper motions. We may now use these in order to learn something of the distances of the stars.

Ceteris paribus, the magnitude of a proper motion will be inversely proportional to the distance of the star. Accordingly, if we use arithmetic means of the proper motions, the results deduced will refer to harmonic means of the distances. There is no special disadvantage in this under certain circumstances; but it is necessary to bear in mind that in using the harmonic mean a great deal of the advantage of taking the average of a large number disappears. The stars which happen to be near us have very great weight in determining the resulting mean; but these are few in number, and are therefore essentially irregularly distributed. The large number of more distant stars on which we should prefer to depend have little effect on the mean.

I have adopted the usual practice of rejecting excessive proper motions; the limit I adopted was to reject those which exceeded four times the mean (excluding themselves). But this does not meet the difficulty; the nominal average of, say, 100 stars is really determined mainly by about 20 stars, and we must bear

this in mind in considering the results about to be given. There is plenty of room for accidental variations in discussing means from 20 stars.

With the notation of § 2,

mean value of $h(r - V\cos\theta)$

$$= \bar{x} = \frac{\int_{-\tau}^{\infty} e^{-x^2}x^2dx + \tau\int_{-\tau}^{\infty} e^{-x^2}xdx}{\int_{-\tau}^{\infty} e^{-x^2}xdx + \tau\int_{-\tau}^{\infty} e^{-x^2}dx}$$

which after some reduction gives

$$\bar{x} = \frac{\frac{1}{2}\, e^{\tau^2}\int_{-\tau}^{\infty} e^{-x^2}dx}{\frac{1}{2} + \tau e^{\tau^2}\int_{-\tau}^{\infty} e^{-x^2}dx}$$

$$\therefore \quad h\bar{r} = \left\{\frac{1}{\sqrt{\pi}}\frac{e^{\tau^2}\int_{-\tau}^{\infty} e^{-x^2}dx}{f(\tau)} + \tau\right\}$$

The following table gives the values of $h\bar{r}$ corresponding to certain values of τ. From these I drew a graph which gave intermediate values with sufficient accuracy.

τ	$h\bar{r}$
−1·0	·565
−0·5	·701
0	·886
+0·2	·977
+0·5	1·134
+0·7	1·252
+1·0	1·449
+1·3	1·669
+1·6	1·908

Now if, out of n stars moving in a certain direction, n_1 belong to drift I., and for them $\tau = \tau_1$, and n_2 belong to drift II., and for them $\tau = \tau_2$, if d_1 d_2 are the mean distances of the stars belonging to the two drifts, and if $\bar{p}$ is the mean proper motion in arc of the n stars,

$$\text{then} \quad \bar{p} = \frac{n_1}{n}\frac{\bar{r}_1}{d_1} + \frac{n_2}{n}\frac{\bar{r}_2}{d_2} \quad . \quad . \quad . \quad (1)$$

Considering stars in a different direction we can get another simultaneous equation, and solve the two for $\frac{1}{d_1}$ and $\frac{1}{d_2}$.

Example.—In region B, consider the proper motions in directions

(α) between 0° and 20°
(β) between 130° and 160°

(α) The analysis showed that we have here 124 drift I. stars and 13 drift II. stars.

Also for drift I., $h\mathrm{V} = 1{\cdot}55$ and the direction of drift I. is 10°, which is also the mean direction of these proper motions.

$$\therefore \quad \tau_1 = 1{\cdot}55 \cos 0^\circ = 1{\cdot}55$$

Similarly,

$$\tau_2 = {\cdot}45 \cos 120^\circ = -{\cdot}22$$

From the graph I found,

corresponding to

$$\tau_1 = 1{\cdot}55, \quad hr_1 = 1{\cdot}87$$
$$\tau_2 = -{\cdot}22, \quad hr_2 = 0{\cdot}81$$

Also the mean proper motion came to be $5''{\cdot}44$, rejecting seven stars whose proper motions exceeded $4 \times 5''{\cdot}44$.

Substituting these values in equation (1),

$$5{\cdot}44'' = \frac{124}{137} \times 1{\cdot}87 \times \frac{1}{hd_1} + \frac{13}{137} \times {\cdot}81 \times \frac{1}{hd_2} \quad . \quad . \quad (2)$$

(β) These are all drift II. stars,

$$\tau_2 = {\cdot}45 \cos 15^\circ = {\cdot}43$$

hence $\quad hr_2 = 1{\cdot}09$

The mean P.M. was found to be $3''{\cdot}07$, rejecting six stars whose P.M.'s exceeded $4 \times 3''{\cdot}07$,

hence $$\frac{1{\cdot}09}{hd_2} = 3''{\cdot}07 \quad . \quad . \quad . \quad . \quad (3)$$

Solving (2) and (3) we find

$$\frac{1}{hd_1} = 3''{\cdot}09 \quad \frac{1}{hd_2} = 2''{\cdot}82 \quad \text{(unit of time = 1 century)}$$

The results of a similar examination of the other regions are

Region.	$\frac{1}{hd_1}$	$\frac{1}{hd_2}$
A	$3''{\cdot}77$	$3''{\cdot}68$
B	$3''{\cdot}09$	$2''{\cdot}82$
C	$2''{\cdot}83$	$3''{\cdot}47$
G	$3''{\cdot}34$	$3''{\cdot}48$

It will be seen that the maximum difference between the numbers in the last two columns is only 25 per cent. of the whole. This perhaps is the only conclusion which can be considered established by the investigation. The figures seem to indicate that where the drift I. stars are nearer to us than the average, there also the drift II. stars are nearer, just as we found that where the stars of one drift were densely distributed, there also the stars of the other drift were dense. However, the main conclusion is that there is no appreciable difference in the mean distances of drift I. and drift II. stars.

Magnitude and Spectral Type.—We cannot, as a rule, pick out an individual star and decide (from its motion) to which drift it belongs. But, for example, in region G we find, on reference to Table VII., that of stars moving in directions

$\theta = 290°–360°$ 257 belong to drift I. and 48 to drift II.
for $\theta = 90°–170°$ 25 „ „ 230 „

We may thus roughly separate out stars typical of the two drifts and examine their characteristics.

The following table shows how stars of different magnitudes are divided between the drifts :—

		Number of Stars of Magnitudes								
		0–3·9	4·0–4·9	5·0–5·9	6·0–6·9	7·0–7·4	7·5–7·9	8·0–8·4	8·4–8·9	Total.
Region A	Drift I.	3	5	28	48	36	21	22	11	174
	Drift II.	1	4	20	48	30	18	26	13	160
Region B	Drift I.	7	10	35	60	47	36	35	19	249
	Drift II.	1	13	33	55	51	40	50	23	266
Region G	Drift I.	6	14	23	63	53	51	47	21	278
	Drift II.	1	6	28	66	44	55	56	25	281
A, B, and G combined	Drift I.	16	29	86	171	136	108	104	51	701
	Drift II.	3	23	81	169	125	113	132	61	707
Carrington	Drift I.	1	0	4	8	14	8	42	77	154
	Drift II.	0	0	6	11	8	16	34	92	167

There is a slightly greater proportion of very faint stars in drift II. in each case. This may be due to the fact that accidental error tends to falsely increase the number of stars apparently in drift II. in this investigation, because, for example, in region B, $\theta = 350° - 30°$ has been taken to typify drift I., and $\theta = 190° - 270°$ for drift II. If a star's motion is purely accidental, there is a much greater chance of its falling within the latter than within the former angular interval.

Most of the very bright stars seem to belong to drift I., but they are few in number altogether. With this possible exception, we may conclude that the drifts do not differ at all as regards the magnitudes of the stars they contain.

As regards *type of spectrum*, the evidence is inconclusive. The proportion of type II. stars seems distinctly higher in drift II. than in drift I. This is true in every region I have examined. On the other hand, the distribution of spectral types seems to be determined mainly by something in no way connected with the drift motions.

The following is the result of the examination of the four most suitable regions :—

Region.	Drift I.			Drift II.			Ratio $\frac{\text{Column 7.}}{\text{Column 4.}}$
	No. of Stars Type I.	No. of Stars Type II.	Ratio $\frac{\text{Type II.}}{\text{Type I.}}$	Type I.	Type II.	Ratio $\frac{\text{Type II.}}{\text{Type I.}}$	
B	95	35	·37	61	45	·74	2·0
G	58	39	·67	41	39	·95	1·4
C	61	23	·38	16	16	1·00	2·6
A	61	66	1·08	36	70	1·94	1·8

The numbers in the last column represent

$$\frac{\text{Ratio of Type II. to Type I. for Drift II.}}{\text{Ratio of Type II. to Type I. for Drift I.}}$$

and show that in each case the proportion of type II. stars is higher in drift II.

We must beware of attaching too much significance to this result, and remember that it applies only to a small part of the sky.

7. *Effect of Errors in the Proper Motions.*

Accidental Error.—The method I have adopted is open to the objection that the stars with very small proper motions, whose directions are presumably not so reliably determined, have the same weight as those with large proper motions. We can, however, take account of small accidental error in the following manner.

If the number of stars having an actual velocity component u be $Ne^{-h^2u^2}$ and the probability of an accidental error x in the observed component proper motion is

$$\frac{g}{\sqrt{\pi}} e^{-g^2x^2}$$

(x being a linear velocity, we confine our attention for the moment to stars at a definite distance).

Then the number of stars having an observed velocity u is

$$N\int_{-\infty}^{\infty} e^{-h^2(u-x)^2} \frac{g}{\sqrt{\pi}} e^{-g^2x^2} dx$$

which reduces to

$$N\frac{g}{\sqrt{g^2+h^2}}\,e^{-\frac{g^2h^2}{g^2+h^2}w^2}$$

Thus the effect of accidental error is simply to change the true drift constant h into an apparent drift constant h_1 given by

$$h_1^2=\frac{g^2h^2}{g^2+h^2}$$

The mean *accidental* error of a Groombridge proper motion appears to be about 0·8″ per century in either co-ordinate (see p. 53). To convert this into linear velocity we must use the values of $\frac{1}{hd}$ found in § 6: the mean value of $\frac{1}{hd}$ is a little over 3″ per century, hence the mean accidental error in the component linear velocity of a star is

$$<\frac{\cdot 8''}{3''}\cdot\frac{1}{h}=\text{about }\frac{1}{4h}$$

But if the probability of an accidental error x be $\frac{g}{\sqrt{\pi}}e^{-g^2}$ the mean error is $\frac{1}{\sqrt{\pi}\cdot g}$

$$\therefore\qquad \frac{1}{4h}=\frac{1}{\sqrt{\pi}\cdot g}$$

$$\text{whence}\qquad \frac{1}{h_1}=1\cdot 09\,\frac{1}{h}\text{ roughly}$$

Thus the effect of accidental error is that if the true value of h is used, the quantities hV given in the preceding sections should be increased by about 10 per cent.

Since g depends on the distance of the stars, this correction is only approximate, and would not hold if the accidental error were large.

Systematic Error.—It is possible that in any region all the proper motions may be liable to a systematic error. For various reasons it can be confidently asserted that in no case does the systematic error much exceed 1″ per century. For instance, let each proper motion be represented by a point whose Cartesian co-ordinates are its components; if there is no systematic error these points will be densest near the origin, but if there is considerable systematic error this will not be so. I have examined the more important regions in this way and found no indication of large systematic error.

Table X. shows the number of proper motions which are greater than 5″ per century in each region in the various directions. This table can hardly be appreciably affected by any admissible value of the systematic error. Nevertheless, the presence of two

TABLE X.—PROPER MOTIONS EXCEEDING 5″.

θ	REGIONS A	B	C	D	E	F	G	Carrington.
5°	29	38	3	2	4	1	7	33
15°	23	31	4	1	2	1	8	30
25°	16	22	8	1	2	1	7	23
35°	10	15	13	2	2	3	4	18
45°	7	10	17	3	2	4	3	14
55°	5	6	18	5	3	5	3	10
65°	3	6	15	6	3	5	5	8
75°	2	6	13	7	2	6	7	7
85°	2	5	7	8	2	5	10	5
95°	2	4	5	10	3	5	11	4
105°	2	4	3	15	4	4	10	6
115°	3	4	4	18	4	4	10	5
125°	3	5	3	18	7	3	11	6
135°	5	6	4	15	8	3	13	7
145°	4	7	3	13	11	2	11	9
155°	5	6	5	9	12	2	7	8
165°	6	4	4	7	16	2	4	6
175°	6	3	3	5	21	2	3	5
185°	4	3	2	5	22	4	4	4
195°	5	3	2	3	19	6	4	5
205°	5	3	1	2	12	10	3	5
215°	5	2	0	1	8	13	3	6
225°	2	2	1	0	6	15	2	4
235°	2	1	1	1	4	12	2	3
245°	3	1	1	2	3	9	4	1
255°	3	0	1	1	2	6	7	1
265°	2	1	1	1	2	3	10	1
275°	1	1	0	0	1	2	10	2
285°	2	1	0	0	0	3	12	2
295°	3	1	0	1	0	3	13	4
305°	4	1	0	0	0	3	15	5
315°	7	2	0	0	1	2	18	8
325°	9	4	0	1	1	1	17	12
335°	14	10	0	1	2	1	16	21
345°	20	22	0	2	4	1	10	27
355°	26	34	1	2	4	1	8	35
Direction drift II.	170°	135°	150°	350°?	?	80°	115°	145°
,, I.	2°	7°	55°	120°	180°	225°	317°	2°

drifts is exhibited very clearly; in fact, more so than when all the proper motions are included. We may therefore feel sure that the phenomena which we have attributed to two drifts are real, and not in any way the effect of systematic error.

A calculation of the positions of the two apices from these large proper motions gives for

Q_1 R.A. 18^h, Decl. $+10°$
Q_2 R.A. 7^h, Decl. $+58°$

in quite good enough agreement with those previously found.

A very important possible systematic error—that due to an incorrect constant of precession—must now be briefly considered.

8. *The Constant of Precession.*

The determination of the proper motion of a star depends on a comparison of observations of its position referred to certain axes at different epochs. In this comparison two unknowns are concerned, viz. the motion of the axes of reference and the motion of the star. The same holds true when we consider a large number of stars; there are two unknowns—the motion of the axes and the systematic motions of the stars. For this reason, in former investigations, determinations of the solar motion and of the constant of precession were necessarily made together. The question arises: Can we attack the fundamental assumptions on which the determination of the solar motion was made without attacking the value of the precession-constant found at the same time?

This is a serious question from the point of view of the present paper, because the constructive and quantitative work in it depends on the correctness of the Struve-Peters precessions. As Dyson and Thackeray pointed out, the area covered by the Groombridge catalogue is not very sensitive to a change in the value of the precession.

If Newcomb's precessions were adopted, the correction to the proper motions would, in the extreme cases (regions C and D), only amount to $0{\cdot}8''$ per century. This would make very little difference in my tables. But may not a much more widely different value of the precession be admissible?

There is an *a posteriori* justification for using the usual value of the precession-constant. We have arrived at the conclusion that the two drifts form a practically homogeneous mixture. If, and only if, this is the case, Airy's method will give the precession. As far as averages are concerned, a homogeneous mixture of two drifts is equivalent to a single drift. Thus the so-called solar motion is an important mathematical vector, but its physical meaning must be modified. We must not be surprised at the discordances which occur in the various attempts to estimate it; it only exists at all because of this roughly uniform arrangement of the drifts, and the precision with which it can be defined depends on this uniformity.

We conclude that the adopted value of the precession is not widely wrong; at the same time, if our theory is correct, we must not trust too far the present methods of calculating it. They are only approximately (and one might almost say accidentally) correct.

9. *Summary and Results.*

The Greenwich-Groombridge and Greenwich-Carrington proper motions strongly support Kapteyn's hypothesis of two star-drifts.

The additional results arrived at in this paper (which, of course, depend on the truth of this hypothesis) are :—

1. The numbers of the stars belonging to the two drifts are nearly equal.

2. One drift is moving relatively to the sun, with a speed between three and four times that of the other.

3. The proportion of stars belonging to each drift is about the same in every part of the sky and at every distance here dealt with.

4. The magnitudes of the stars belonging to the two drifts are about the same.

5. The spectral types of the stars seem to differ to some extent in the two drifts. But the evidence is not conclusive.

6. The results apply to all stars down to magnitude 9·5.

Royal Observatory, Greenwich:
1906 *August.*

On the Effects of Radiation on the Motion of Comets (Second Note). By H. C. Plummer, M.A.

1. Professor Poynting has recently pointed out to me the existence of an additional term in the forces which are due to the action of solar radiation on a small black body. The term has no effect when the motion of the particle is circular, but it becomes necessary to amend the theory which was given in §§ 5–8 of my former paper (*Monthly Notices*, vol. lxv. p. 232). This is due to the fact that when the body is moving with a radial velocity $\frac{dr}{dt}$ the light pressure is altered in the ratio $1 : 1 - \frac{1}{U} \cdot \frac{dr}{dt}$ where U is the velocity of light. The effect of the radial component of the motion was before omitted.

2. The radial accelerations of the particle are

(i.) that due to gravitation,

$$R_g = -\frac{\mu}{r^2}$$

(ii.) that caused by a "static" pressure due to radiation,

$$R_s = +\frac{\mu'}{r^2}$$

(iii.) that caused by a "kinetic" pressure due to radiation (this is the term now introduced),

$$R_k = -\frac{1}{U}\frac{dr}{dt}\cdot\frac{\mu'}{r^2}$$

There is also a tangential retardation Tv/r^2, which may be resolved into

(iv.) a radial component

$$R_t = -\frac{T}{r^2}\cdot\frac{dr}{dt}$$

(v.) a transversal component

$$V_t = -\frac{T}{r}\cdot\frac{d\theta}{dt}$$

Now if S is the solar constant at the Earth's distance b from the Sun, and α is the radius of the spherical particle of density ρ,

$$T = Sb^2/4U^2\alpha\rho \quad . \quad . \quad . \quad . \quad (1)$$
$$\mu' = 3Sb^2/4U\alpha\rho = 3UT \quad . \quad . \quad . \quad (2)$$

Hence the complete radial component is

$$\Sigma R = -\frac{\mu-\mu'}{r^2} - \frac{4T}{r^2}\cdot\frac{dr}{dt} \quad . \quad . \quad . \quad (3)$$

3. One equation of motion remains as before

$$\frac{d}{dt}\left(r^2\frac{d\theta}{dt}\right) = rV_t = -T\frac{d\theta}{dt}$$

or on integration

$$r^2\frac{d\theta}{dt} \equiv h = \frac{T}{c}(1 - c\theta) \quad . \quad . \quad . \quad (4)$$

where c is a small constant of the same order as T. The other equation of motion is

$$\frac{d^2r}{dt^2} - r\left(\frac{d\theta}{dt}\right)^2 = \Sigma R$$

which becomes on eliminating t and putting $u = 1/r$

$$h^2u^2\left(\frac{d^2u}{d\theta^2} + u\right) + hu^2\frac{dh}{d\theta}\cdot\frac{du}{d\theta} + \Sigma R = 0$$

By (3) and (4) this is reduced to

$$\frac{d^2u}{d\theta^2} + u + \frac{3c}{1-c\theta}\frac{du}{d\theta} = \frac{(\mu-\mu')c^2}{(1-c\theta)^2T^2} \quad . \quad . \quad (5)$$

which differs from the corresponding equation of the former paper by the addition of the third term on the left.

4. Neglecting second-order terms we replace (5) by

$$\frac{d^2u}{d\theta^2} + 3c\frac{du}{d\theta} + u = (\mu - \mu')c^2T^{-2}(1 + 2c\theta)$$

the integral of which, to the same order of approximation, is

$$u = (\mu - \mu')c^2T^{-2}\{1 + 2c\theta + e\cos(\theta - \gamma) - \tfrac{3}{2}ce\theta\cos(\theta - \gamma)\} \quad (6)$$

Let the osculating orbit at the point θ be

$$u = l^{-1}\{1 + e'\cos(\theta - \gamma')\}$$

Since r, $\frac{dr}{dt}$ and $\frac{d\theta}{dt}$ are the same for the two curves, u, $\frac{du}{d\theta}$ and h are also the same. Therefore by (4)

$$l = \frac{h^2}{\mu - \mu'} = \frac{T^2}{c^2}\frac{(1 - c\theta)^2}{\mu - \mu'} \quad . \quad . \quad . \quad (7)$$

where θ is to be considered constant at the point. Hence we have

$$\frac{uT^2}{(\mu - \mu')c^2} = 1 + 2c\theta + e\cos(\theta - \gamma) - \tfrac{3}{2}ce\theta\cos(\theta - \gamma)$$
$$= (1 + 2c\theta)\{1 + e'\cos(\theta - \gamma')\}$$

and also $\frac{du}{d\theta}$ the same for both expressions, provided that in the second we regard $c\theta$ as constant. Hence, still neglecting terms beyond the first order,

$$e'\cos(\theta - \gamma') - e\cos(\theta - \gamma) = -\tfrac{7}{2}ce\theta\cos(\theta - \gamma)$$
$$e'\sin(\theta - \gamma') - e\sin(\theta - \gamma) = -\tfrac{7}{2}ce\theta\sin(\theta - \gamma) + \tfrac{3}{2}ce\cos(\theta - \gamma) - 2c$$

From these can be derived

$$\delta e = -\tfrac{7}{2}ce\theta + \tfrac{3}{4}ce\sin 2(\theta - \gamma) - 2c\sin(\theta - \gamma) \quad . \quad (8)$$
$$e\,.\,\delta\gamma = -\tfrac{3}{2}ce\cos^2(\theta - \gamma) + 2c\cos(\theta - \gamma) \quad . \quad . \quad (9)$$

where the increments refer to the osculating ellipse as compared with the ellipse obtained by suppressing the accents on e and γ.

5. The secular changes in the elements can now be deduced from (7), (8), and (9) by considering the effect of a complete revolution. The result of thus removing the periodic variations is clearly

$$\Delta l/l = -4\pi c \quad . \quad . \quad . \quad . \quad (10)$$
$$\Delta e/e = -7\pi c \quad . \quad . \quad . \quad . \quad (11)$$
$$\Delta\gamma = 0 \quad . \quad . \quad . \quad . \quad . \quad (12)$$

Thus the result of the additional term in the action of solar radiation is by (11) to increase the secular change in the eccentricity in the ratio of 7 : 4.

Further,

$$\frac{\Delta a}{a} = \frac{\Delta l}{l} + \frac{2e\Delta e}{1 - e^2} = -\frac{4 + 10e^2}{1 - e^2}\cdot\pi c \quad . \quad . \quad (13)$$

and

$$\frac{\Delta n}{n} = -\tfrac{3}{2} \cdot \frac{\Delta a}{a} = \frac{2+5e^2}{1-e^2} \cdot 3\pi c \quad . \quad . \quad . \quad (14)$$

The effect of the new term is thus to increase the change in the mean motion in the ratio

$$2+5e^2 : 2+2e^2$$

so that, with the same notation and constants as before, equation (1) of the previous paper becomes

$$\rho a = 4{\cdot}6 \times 10^{-15} \frac{(2+5e^2)(1-e)^{\frac{1}{2}}}{(1+e)^{\frac{3}{2}}} \cdot \frac{b^2}{q^2} \cdot \frac{1}{\Delta n} \quad . \quad (15)$$

6. For Encke's Comet, $e = 0{\cdot}85$, we now obtain

$$\rho a = 0{\cdot}0093 \text{ gm./cm.}^2 ; \ \mu'/\mu = 0{\cdot}008$$

These figures do not differ very materially from those previously found. They are of the same order, and equally fail to substantiate the suggestion that an acceleration in the mean motion of Encke's Comet can be explained in this particular way. This conclusion, though disappointing, is only what might have been anticipated. Of the forces enumerated in § 2, the components (iii.), (iv.), and (v.), as compared with (ii.), the direct radiation pressure, are of the same order as cometary velocities compared with the velocity of light. Their effect is therefore naturally far smaller, and can only become adequate to explain a conspicuous inequality when the gravitation constant itself is effectively diminished by the back pressure due to radiation. The forces which depend on the velocity of the comet are dissipative, as Professor Poynting notices, and therefore cumulative in effect. But though their influence may thus become of the greatest importance by the lapse of time, it is the conservative component of which evidence must first be expected from direct observation.

7. Since my earlier paper the question of evidence on this point has advanced, and we now know that as long ago as 1891 the late Dr August Svedstrup had been led by his labours on the motion of Comet 1886 I. to the conclusion that the constant of solar attraction was in this case effectively diminished. The result is the more remarkable that the author seems to have been uninfluenced by any hypothesis as to the action of radiation. In place of the Gaussian constant k, he found it necessary to use (A.N., 4062, p. 98)

$$\mathrm{K} = k(1 - 0{\cdot}0000415)$$

which gives

$$\mu'/\mu = 1 - \mathrm{K}^2/k^2 = 0{\cdot}000083$$

Hence

$$a\rho = \frac{3}{4} \cdot \frac{\mathrm{S}}{\mathrm{U}} \cdot \frac{b^2}{\mu} \cdot \frac{10^6}{83}$$

$$= 0{\cdot}89 \text{ gm./cm.}^2$$

if we adopt $S = 1{\cdot}75 \times 10^6$. This shows that if ρ is about 6, or something like the mean density of the Earth, the diameter of the particles composing the comet is of the order of 3 mm. This result confirms the hypothesis as to the physical constitution of comets on which this investigation rests, and supplies the first numerical estimate of the size of the meteoric bodies which constitute a cometary swarm.

8. Dr Svedstrup's discovery amply justifies the statement before made, that "it is unwarrantable to *assume* that the mean motion and the mean distance are related and not independent elements." This must be borne in mind in all future discussions of the definitive orbits of comets; otherwise a valuable means of gaining an insight into the constitution of these bodies will be neglected. It now appears exceedingly probable that in this direction lies the clue to the explanation of the anomalous motion of Encke's Comet, as suggested in § 3 of my earlier paper.

University Observatory, Oxford:
1906 *August* 18.

Note on a Mechanical Solution of Kepler's Equation.

By H. C. Plummer, M.A.

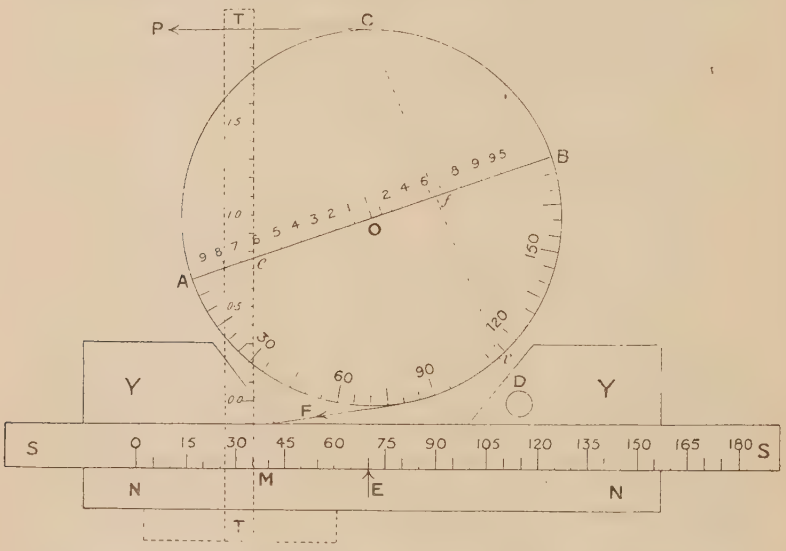
This note refers to the method of obtaining an approximate solution of Kepler's equation which was proposed by Dr Rambaut (*Monthly Notices*, vol. l. p. 301), and was subsequently described independently by myself (*Monthly Notices*, vol. lvi. p. 317), and again later by Herr B. Gonggrijp (*Ast. Nach.*, 3720). The principle of the method, which is based on the use of a trochoid, is to be found in Newton's *Principia*, book i. prop. 31.

Quite recently Dr Rambaut has proposed a new method which combines great mathematical elegance with considerable practical advantages (*Monthly Notices*, vol. lxvi. p. 519). His paper suggests that it may not be out of place if I describe some modifications in the application of the trochoid principle which occurred to me, by a curious coincidence, about the same time.

In the accompanying figure, A B C is a flat circular disc of suitable radius, mounted on a board so as to be capable of turning about its centre O. Fixed to the board and flush with the surface of the disc are wooden pieces Y Y, N N, which serve as guides to a scale S S, which, with bevelled edges, can slide between them. The point E, marked with a pointer, is the foot of the perpendicular from O on the lower edge of the scale. Attached near B, and passing round the edge of the disc, is a thin flexible metallic tape F, the other end of which is attached to the end of the scale S S. This tape is kept taut by another tape B C P passing round the disc in the opposite direction and fastened to a weak spring or, much better, to a hanging weight.

The rotation of the disc thus depends on and can be measured by the motion of the scale S S, a part of which, in length equal to half the circumference of the disc, is divided into 180° in such a way that the origin coincides with the point E when the radius O A is perpendicular to the scale. Then clearly the pointer E at any time marks the angle (< 180°) through which the disc has been turned from its initial position.

The initial radius O A is divided in fractions of itself (as shown, in tenths), so that a length O*e* can be taken corresponding to the eccentricity, the whole radius being unity. The lower edge of the guide N N is parallel to the scale S S, so that the edge of a T-square



T T can be kept perpendicular to the scale. The instrument can now be used to find an approximate solution of Kepler's equation, according to the following rule:—

Place the edge of the T-square at the reading M *on the scale* S S. *Move the scale and T-square together till the point e on* O A *which marks the eccentricity e falls on the edge of the T-square. Then read the position of the pointer* E *on the scale. The reading is the eccentric anomaly* E *corresponding to the mean anomaly* M.

For clearly

$$E - M = e \sin E \quad . \quad . \quad . \quad . \quad (1)$$

If M *lies between* 180° *and* 360°, *use the instrument according to the above rule to find* 360° − E *from* 360° − M.

The figure is drawn for $e = 0{\cdot}65$, showing M = 35°·0 and E = 70°·0. The rolling disc of the device formerly described, with its troublesome manipulation and liability to error, is now eliminated. The present instrument is nearly as simple to use as a slide-rule, and

it has the great advantage of being direct-reading. Two straight, uniformly divided scales O A and S S alone are necessary; and the accuracy of reading can be increased by placing a vernier instead of a simple pointer at E, and attaching another vernier to the T-square at M. The surface of the scale S S should be very slightly above the level of Y Y, N N and the disc. The T-square can then be kept at a fixed reading with one hand while the scale is being moved by the other. It would be still better if the head of the T-square moved along a groove in the scale itself. In such details of construction there is naturally much room for variety, but it is possible to make an instrument on these general lines which is both simple and efficient.

There are, however, three additional scales shown in the figure which may be briefly explained. The first is on the edge of the T-square, which is graduated in fractions of the radius of the disc. The origin of this scale is on a level with the lowest point of the disc. It is thus possible to read directly at the point e the height of this point above the lowest point of the disc, and this reading clearly corresponds to the quantity

$$1 - e\cos \mathrm{E} = \frac{r}{a} = \frac{d\mathrm{M}}{d\mathrm{E}} \quad . \quad . \quad . \quad (2)$$

This is the ratio of the radius vector to the mean distance, but its more important significance arises from the fact that it is the ratio of corresponding increments of the mean and eccentric anomalies. Thus when an approximate E (as found by mechanical means or otherwise) is known, and is found to lead to an error dM in M, the approximate error in the assumed E is found by dividing dM by the reading on the edge of the T-square at the point where it is crossed by the radius O A of the disc. In the case represented in the figure this reading should be 0·778.

Up to this point we have considered only practical modifications in applying the principles of a device formerly described. The graduation of one semicircle and the unequally divided scale on the radius O B, as shown in the figure, are due to an addition which, whatever its value, appears to be new. The object of this is to obtain by a direct reading an approximate value of the true anomaly. The graduations in the radius O B (numbered in the figure for 10 e) are made in such a way that corresponding to any eccentricity e a point f can be read off such that

$$Of/\mathrm{OB} = \tan \tfrac{1}{2}\phi \quad \text{where} \quad \sin\phi = e$$

A thread C D is attached to the point C, the highest point of the disc. The disc being in the position already found, the thread is stretched over the point f and crosses the edge of the disc at v. Then the angle A O v or v is such that

$$\tan \tfrac{1}{2}v = \sqrt{\frac{1+e}{1-e}}\tan \tfrac{1}{2}\mathrm{E} \quad . \quad . \quad . \quad (3)$$

so that v is the true anomaly corresponding to the mean anomaly M.

The geometrical principle on which this very simple device is based need not be proved here.* An accurate value of the true anomaly can only be found by calculation from an accurate value of E. But there may be cases in which only a rough value is required, and in any case a value of this kind, when so easily obtained, may be useful in preventing any serious slip in the calculation from being overlooked. In the case represented in the figure the value of v should be $113^{\circ}{\cdot}3$.

The complete instrument thus provides the means of finding, for any value of the eccentricity between 0·1 and 0·9, approximate values of the eccentric and true anomalies and of the radius vector, when the mean anomaly or time is given. It thus solves in an approximate manner the whole problem of elliptic motion as expressed by the relations (1), (2), and (3), and the ease with which this can be done suggests that the instrument may have some slight educational as well as practical value.

1906 *September* 18.

Solar Parallax Papers. No. 5.

Examination of the Photographic Places of Stars published in the Paris Eros Circulars. By Arthur R. Hinks, M.A.

1. In a preceding paper, *Solar Parallax Papers, No. 4* (*Monthly Notices*, 1906 June, lxvi. p. 481), it was shown that the photographic right ascensions of *repère* stars, published in Paris circulars 10 and 11, had little if any magnitude equation special to each observatory (with one exception); and the conclusion was drawn that it is unlikely that they have an absolute magnitude equation common to all. This applies only to stars down to a magnitude little fainter than 9th. In the present paper I propose to extend the search for photographic magnitude equation to the declinations of the *repère* stars, and to the fainter stars in both co-ordinates; and to look for systematic errors other than those depending on magnitude, so far as they can be detected by intercomparison of published results.

2. The stars measured upon Eros plates in accordance with M. Loewy's programme are divided into three classes:—

i. The *étoiles de repère*, extending right up to the edges of plates 2° square, and generally in the outer regions rather than near the centre.

ii. The *étoiles de comparaison*, which have been used as comparison stars in visual micrometric observations. They lie mostly in a narrow belt along the track of the planet, and extend right up to two edges of the plate.

* The principle is the same as that employed in "A Method of Mechanically Compensating the Rotation of the Field of a Siderostat" (*M.N.*, vol. lxi. p. 402). See § 2 for a proof which can be adapted without difficulty to the present case, and §§ 8 and 9 for an examination of the geometry of the principle.

Table I.

Repère Stars. Magnitude Equation in Photographic Declinations.

List I.

Magnitudes.	Paris minus— Alg.	Bord.	Cat.	Green.	Hels.	North.	S. Fern.	Toul.	Mean of 7.
−6·2	(5) −″·45	(2) +″·14	(3) +″·30	(3) −″·05	(3) +″·36	(3) +″·06	(5) −″·08	(5) −″·51	(24) −″·03
6·3 – 6·9	(6) − 17	…	(2) + 4	(5) + 7	(4) + 16	(5) + 22	(6) + 1	(6) − 3	(28) + 07
7·0 – 7·4	(15) − 32	(7) + 4	(9) + 12	(14) + 1	(9) − 12	(12) − 10	(15) − 10	(15) − 9	(81) − 4
7·5 – 7·9	(27) − 10	(12) + 11	(21) + 11	(20) − 1	(15) − 2	(20) − 3	(27) + 1	(27) − 7	(142) + 1
8·0 – 8·4	(60) 0	(24) + 5	(39) + 7	(47) + 3	(22) + 5	(33) 0	(53) + 13	(53) + 4	(271) + 3
8·5 – 8·8	(70) + 4	(40) 0	(43) + 3	(60) 0	(38) − 9	(46) − 2	(66) + 9	(67) + 1	(360) + 1
8·9 – 9·2	(45) + 7	(24) + 5	(31) + 13	(35) − 3	(20) − 4	(33) 0	(44) + 9	(44) + 5	(231) + 4
9·3 –	(12) + 21	(9) + 4	(8) + 11	(8) 0	(10) + 4	(7) − 3	(13) + 24	(13) + 20	(68) + 10
Totals	(240) − ·01	(118) + ·04	(156) + ·08	(192) ·00	(121) − ·02	(159) − ·01	(229) + ·05	(230) + ·01	…

List II.

−6·2	(5) ·23	(2) + ·56	(4) + ·53	(3) − ·09	(5) + ·14	(3) + ·40	(5) + ·26	(3) + ·43	(25) + ·30
6·3 – 6·9	(7) − 12	(4) − 11	(7) − 16	(4) − 6	(3) + 31	(1) − 19	(7) + 29	(5) + 24	(31) − 1
7·0 – 7·4	(8) − 10	(4) − 19	(5) + 16	(8) + 1	(8) + 12	(2) − 10	(8) − 5	(7) − 7	(42) 0
7·5 – 7·9	(17) + 2	(6) − 8	(12) + 3	(12) + 8	(12) − 2	(5) − 20	(17) − 4	(11) − 4	(75) − 2
8·0 – 8·4	(18) − 4	(13) 3	(15) 0	(16) − 10	(16) + 5	(9) + 10	(25) + 2	(18) − 2	(112) 0
8·5 – 8·8	(24) + 6	(15) + 10	(16) − 8	(16) + 3	(18) + 3	(8) − 4	(24) + 5	(17) + 6	(114) + 3
8·9 – 9·2	(3) + 14	(3) − 16	(3) − 13	(3) + 6	(3) + 23	(1) + 27	(3) − 3	(3) − 12	(19) − 1
9·3	…	…	…	…	…	…	…	…	…
Totals	82) − ·01	(47) ·00	(62) + ·01	(62) − ·01	(65) + ·06	(29) + ·02	(89) + ·04	(64) − ·01	…

Note.—The number of stars contributing to each mean is given in brackets before it.

iii. The *étoiles du carré de* 20′, including all stars on the plate within a square of twenty minutes of arc, having the planet at the centre—the planet being always near the centre of the plate.

Unfortunately for the study of systematic errors, the places of the stars in these three categories have been published in three different ways in the Paris Circulars.

For the *étoiles de repère*, Tableau I. gives the mean place concluded from the whole number of plates.

For the *étoiles de comparaison*, Tableau III. gives, in some cases, collections of the separate individual results, but without indication of the plates from which they are derived; in other cases, means only.

For the *étoiles du carré* the Tableau II. gives the complete individual results, plate by plate.

It is therefore impossible to separate completely the proceeds of one plate from the rest, and this is a grave obstacle to the proper investigation of the results. For the present we are restricted to the study of mean places (except for a small central region). We shall therefore, in what follows, draw no distinction between classes II. and III., and shall refer to all the fainter stars as "comparison stars."

3. *Magnitude Equation in Photographic Declinations. Repère Stars.*—The comparison of the photographic declinations of *repère* stars, grouped according to magnitude, is presented in the same form as was the comparison of photographic R.A.'s in my last paper (*loc. cit.*, p. 483), but with the addition of a column for Paris minus mean of seven (Algiers being excluded).

For list I. stars, Algiers has a considerable magnitude equation in declination, as it had in R.A. San Fernando shows no very evident sign of it; but Toulouse has a well-marked magnitude equation, which is unexpected and very interesting. Bordeaux, Catania, Greenwich, Helsingfors, Northfield, and Paris show no serious divergence one from another, and we conclude that they are sensibly free from magnitude equation.

The comparison with list II. extends at present only to R.A. $3^h\ 17^m$ (corresponding to 1901 January 25). Up to that point there is little evidence of magnitude equation, even for Algiers.

4. *Magnitude Equation in Visual Declinations. Tucker's System.*—This comparison follows very closely on the lines of the preceding.

Comparison with the mean of six observatories (excluding Algiers, Toulouse, and San Fernando), and with the six individually, shows little or no evidence of any magnitude equation in Tucker's declinations, unless possibly for the brightest stars, for which material is insufficient.

And the various systems adopted by different observatories show no certain trace of relative magnitude equation when compared with Tucker.

We may conclude that the meridian circle declinations in general are practically free from personality depending on magnitude —an interesting and somewhat unexpected conclusion.

Table II.

Repère Stars. Examination of Tucker's Declinations for Magnitude Equation.

List I.	System T. minus— Alg.	Bord.	Cat.	Green.	Hels.	North.	Paris.	S. Fern.	Toul.	Mean of 6.
−6·2	(7) −″·42	(2) +″·18	(5) −″·07	(4) +″·13	(4) +″·32	(5) +″·22	(5) +″·12	(7) −″·06	(7) −″·33	(25) +″·14
6·3−6·9	(7) − 17	...	(2) + 5	(5) + 11	(4) + 21	(5) + 27	(6) + 7	(7) + 12	(7) + 9	(22) + 15
7·0−7·4	(20) − 23	(7) + 7	(13) + 10	(18) + 7	(12) 0	(14) + 3	(15) + 6	(20) − 4	(20) − 7	(79) + 6
7·5−7·9	(35) − 6	(12) + 8	(25) + 18	(22) + 2	(17) 0	(28) − 2	(27) − 2	(35) − 4	(35) − 4	(131) + 4
8·0−8·4	(58) 0	(24) + 2	(44) + 6	(49) + 2	(26) + 4	(36) · 1	(53) · 2	(58) + 12	(57) + 1	(232) + 2
8·5· 8·8	(74) + 6	(41) + 3	(51) 0	(60) + 1	(39) · 1	(47) + 3	(66) + 2	(77) + 12	(77) + 2	(304) + 1
8·9−9·2	(54) + 10	(24) + 2	(38) + 9	(38) 0	(23) − 9	(35) − 6	(44) − 2	(55) + 9	(55) + 3	(202) − 1
9·3−	(12) + 13	(9) + 4	(9) + 17	(8) + 2	(10) − 1	(8) − 1	(13) − 4	(14) + 17	(14) + 16	(57) + 2
Totals	(267) ·00	(119) + ·04	(187) + ·07	(204) + ·02	(135) ·00	(178) + ·01	(229) ·00	(273) + ·08	(272) + ·01	...
List II.										
−6·2	(5) − ·51	(2) − ·08	(4) + ·10	(3) + ·09	(5) + ·06	(3) + ·08	(5) − ·28	(5) − ·02	(3) − ·18	(22) − ·01
6·3−6·9	(7) − 25	(4) − 25	(7) · 25	(4) − 20	(3) + 3	(1) − 75	(7) − 13	(7) + 16	(5) − 39	(26) − 20
7·0−7·4	(9) − 17	(4) − 21	(6) + 17	(9) − 5	(8) + 2	(2) − 14	(8) − 5	(8) − 12	(7) − 14	(37) − 2
7·5−7·9	(17) − 8	(6) − 23	(12) − 10	(12) + 1	(12) · 6	(5) − 23	(17) − 14	(17) − 18	(11) − 12	(64) − 11
8·0−8·4	(18) − 20	(13) − 21	(15) · 15	(16) − 22	(16) + 2	(9) − 2	(19) − 16	(19) · 13	(18) − 16	(88) − 13
8·5−8·8	(24) 0	(15) 0	(16) − 15	(16) − 4	(19) − 3	(8) · 7	(24) − 6	(24) − 1	(17) − 2	(98) − 6
8·9−9·2	(3) − 13	(3) − 43	(3) − 41	(3) − 21	(3) − 1	(1) + 3	(3) − 27	(3) − 30	(3) − 39	(16) − 23
9·3−	...	...	...	...	...	...	...	...	...	...
Totals	(83) − ·13	(47) − ·09	(63) − ·12	(63) − ·09	(66) − ·01	(29) − ·09	(83) − ·13	(83) − ·08	(64) − ·14	...

But the comparison of Tucker with the photographic results of Algiers and Toulouse exhibits the magnitude equation in the latter previously discovered.

The results of the comparison with list II. are too irregular to be of much service.

5. *Magnitude Equation in Photographic R.A. Comparison Stars.*—In extending to the fainter stars our search for magnitude equation, we meet with the difficulty that the magnitudes assigned to these stars at different observatories are discordant. For example, on tabulating several hundred comparison stars, we find that the magnitudes assigned at Paris are greater in the mean by $1^m{\cdot}6$ than those assigned to the same stars at Bordeaux, and greater by $0^m{\cdot}6$ than those assigned to the same stars on Catania plates, though these were measured at Paris. It would seem that the Paris "magnitudes" are really intensities of image on the plate, unreduced to any photometric scale; excellent, therefore, for investigating magnitude equation on plates treated individually, but of less merit when combined into means.

Because of these discordances in the assigned magnitudes, one must discuss the differences—Paris—Bordeaux, for example—in duplicate, once with the Paris magnitudes, and a second time with the Bordeaux. We find below the classification according to magnitude of the differences Paris—Bordeaux, and also of the differences Bordeaux—Paris (the same quantities with signs changed). The first classification is by Paris magnitudes, and the compartment $9^m{\cdot}9$–$10^m{\cdot}5$ contains 17 stars. The second division is by Bordeaux magnitudes, and the same compartment contains 101 stars. A result still more curious occurs in the comparison between Paris and San Fernando. The last group $12^m{\cdot}2$ and fainter contains 158 stars in the first classification by Paris magnitudes. On repeating it with the San Fernando magnitudes, all but two of these stars are thrown back into preceding groups. It is clear that there will be difficulties in expressing an error as a function of the magnitude.

6. The results here discussed cover nearly the same stretch of the planet's path as does list I. of the *repère* stars, and depend largely, though not entirely, upon stars common to the Paris and other series. The comparison is therefore somewhat incomplete. Quite early in the work it was realised that definitive corrections could not be derived from this material as it stands; that we should have to repeat the whole work in greater detail, and that it was not worth while to spend time in making the present preliminary discussion absolutely complete.

Photographic places of comparison stars have been published in Circulars 10 and 11 by Paris, Bordeaux, Catania, San Fernando, Toulouse, and Algiers. Each one of these six observatories has been compared with each of the others. But it is hardly necessary to give here all the results. It will be sufficient if we give the comparisons of Paris, San Fernando, and Algiers with one another and with the others.

The first set shows that there is no serious discordance between Paris and others, with the exception of San Fernando. To see if the apparent magnitude equation in San Fernando is real, we compare it with each of the others. They are unanimous in showing a relative magnitude equation, approximately linear, of about $0^s.009$ per magnitude. But we must notice that this is of opposite sign to that suspected in the *repère* stars. We shall return to this point in § 8.

The comparison of others with Algiers shows that the extraordinary magnitude equation of the *repère* stars is maintained through the fainter magnitudes. For stars of the sixth magnitude the Algiers R.A. is small by about $0^s.03$; for the twelfth magnitude it is large by $0^s.08$—the whole range equal to about $1''$.

Table III.

Comparison Stars. Magnitude Equation in Photographic R.A.'s.

Paris minus—

m	Bord. s	Cat. s	S. Fern. s	Toul. s	Alg. s
– 9·2	(9) – ·001	(14) + ·016	(20) + ·009	(21) + ·001	(20) – ·005
9·3 – 9·8	(8) – 10	(14) – 3	(14) – 5	(17) – 9	(16) – 33
9·9 – 10·5	(17) – 2	(24) + 6	(46) + 9	(50) + 6	(42) – 32
10·6 – 11·2	(36) + 1	(73) + 2	(78) + 12	(79) 0	(71) – 48
11·3 – 12·1	(56) – 1	(153) – 4	(139) + 22	(129) + 6	(105) – 67
12·2 –	(93) – 7	(153) + 2	(158) + 21	(148) 0	(80) – 88
Totals	(219) – ·004	(431) + ·001	(455) + ·017	(444) + ·002	(344) – ·056

San Fernando minus—

m	Bord. s	Cat. s	Paris. s	Toul. s	Alg. s
– 9·2	(28) – ·007	(50) – ·001	(91) – ·008	(98) – ·004	(93) – ·031
9·3 – 9·8	(39) – 12	(50) – 16	(68) – 14	(68) – 8	(61) – 68
9·9 – 10·5	(72) – 20	(112) – 21	(113) – 17	(109) – 15	(101) – 86
10·6 – 11·2	(57) – 23	(112) – 24	(135) – 21	(96) – 12	(74) – 92
11·3 – 12·1	(16) – 39	(32) – 27	(46) – 34	(38) – 24	(24) – 108
12·2 –	(1) + 20	(1) + 40	(2) – 39	(4) + 2	(3) – 138
Totals	(213) – ·019	(357) – ·019	(455) – ·017	(413) – ·011	(356) – ·072

Algiers minus—

m	Bord. s	Cat. s	Paris. s	S. Fern. s	Toul. s
– 9·2	(25) + ·023	(41) + ·031	(76) + ·022	(79) + ·032	(81) + ·026
9·3 – 9·8	(31) + 57	(40) + 63	(63) + 50	(59) + 60	(63) + 54
9·9 – 10·5	(26) + 63	(37) + 83	(40) + 62	(46) + 66	(45) + 57
10·6 – 11·2	(42) + 66	(72) + 79	(76) + 69	(86) + 95	(84) + 82
11·3 – 12·1	(39) + 73	(39) + 87	(81) + 77	(80) + 96	(85) + 82
12·2 –	(2) + 30	(5) + 112	(8) + 89	(6) + 90	(10) + 88
Totals	(155) + ·062	(234) + ·071	(344) + ·056	(356) + ·072	(368) + ·062

Tables similar to the above for Bordeaux, Catania, and Toulouse show few features of interest. For Bordeaux minus others, the numbers fall slightly to the third group, and rise again. For Catania and Toulouse there is a slight upward tendency at the beginning and end. But the whole range is small, equivalent to about $0''\cdot2$; and, in light of what will follow, we cannot attribute it to magnitude equation.

7. *Magnitude Equation in Photographic Declinations. Comparison Stars.*—The procedure followed in discussing the R.A.'s has been repeated for the declinations without change. A selection of the more interesting results is given in Table IV.

Paris, Bordeaux, and Catania agree fairly well. We can hardly look upon the differences Paris – Bordeaux as significant of magnitude equation until we have examined the effects of the difference of adopted places in Bordeaux, which is a very incomplete series.

Toulouse diverges strongly from these three accordant series, and has a well-marked nearly linear magnitude equation in the declinations of comparison stars of about $0''\cdot1$ per magnitude.

San Fernando is a difficult case. We may summarise thus:—

Paris —San Fernando (P. magnitudes) shows a very slight M.E.
Bordeaux —San Fernando (B. magnitudes) shows decided M.E.
Catania —San Fernando (C. magnitudes) shows large M.E. of more than $0''\cdot1$ per magnitude.

On the other hand:

San Fernando —Paris (S.F. magnitudes) shows no M.E. whatever.
,, —Bordeaux (S.F. magnitudes) shows none, or, at most, traces of a non-linear M.E.
,, —Catania (S.F. magnitudes) shows a very small and uncertain M.E.

The discordance between the comparisons Catania—San Fernando and San Fernando – Catania, that is to say, between the figures grouped differently according as one accepts the Catania or San Fernando magnitudes, shows that no definitive corrections can be derived from the present work. The magnitude equation, if any, is doubtless entangled with other systematic errors of Catania or San Fernando, for which see later, § 14.

Algiers shows a considerable magnitude equation, as before, but it is not nearly so variable in the fainter magnitudes as in the brighter.

Table IV.

Comparison Stars. Magnitude Equation in Photographic Declinations.

	Paris minus—				
	Bord.	Cat.	S. Fern.	Toul.	Alg.
– 9·2	(8) + ″·03	(14) – ″·03	(21) – ″·09	(21) + ″·11	(20) – ″·04
9·3 – 9·8	(5) 0	(14) – 5	(14) + 22	(19) + 28	(18) + 22
9·9 – 10·5	(18) + 2	(25) + 1	(47) + 24	(48) + 28	(42) + 33
10·6 – 11·2	(37) + 9	(72) + 3	(78) + 20	(81) + 29	(68) + 29
11·3 – 12·1	(59) + 12	(153) + 2	(138) + 30	(126) + 37	(103) + 39
12·2 –	(93) + 4	(152) + 1	(157) + 28	(149) + 45	(93) + 46
Totals	(220) + ·07	(430) + ·01	(455) + ·26	(444) + ·36	(344) + ·35

	San Fernando minus—				
	Bord.	Cat.	Paris.	Toul.	Alg.
– 9·2	(29) – ″·10	(50) – ″·15	(93) – ″·24	(97) – ″·05	(93) – ″·03
9·3 – 9·8	(38) – 13	(52) – 13	(67) – 25	(69) + 4	(61) + 11
9·9 – 10·5	(73) – 24	(111) – 25	(119) – 23	(108) + 14	(101) + 18
10·6 – 11·2	(59) – 17	(113) – 29	(129) – 30	(98) + 16	(74) + 23
11·3 – 12·1	(15) – 16	(31) – 24	(45) – 25	(38) + 7	(24) + 25
12·2 –	(1) – 20	(1) – 10	(2) + 7	(4) + 7	(3) + 4
Totals	(215) – ·18	(358) – ·23	(455) – ·26	(414) + ·08	(356) + ·13

	Toulouse minus—				
	Bord.	Cat.	Paris.	S. Fern.	Alg.
– 9·2	(21) – ″·11	(45) – ″·20	(82) – ″·22	(74) + ″·05	(77) ″·00
9·3 – 9·8	(32) – 18	(44) – 21	(74) – 31	(70) – 5	(65) + 8
9·9 – 10·5	(72) – 32	(129) – 29	(129) – 35	(127) – 16	(112) – 1
10·6 – 11·2	(55) – 35	(98) – 35	(123) – 42	(113) – 8	(94) + 7
11·3 – 12·1	(12) – 61	(15) – 28	(36) – 56	(30) – ·9	(20) 0
12·2 –	...	...	...	...	...
Totals	(192) – ·30	(331) – ·29	(444) – ·36	(414) – ·08	(368) + ·03

8. *Summary of Results for Photographic Magnitude Equation.* —We may now bring together the results for the *repère* and comparison stars.

Regrouping the *repère* stars into larger magnitude groups, to avoid the roughness due to paucity of numbers, we have the following table for the comparison of Toulouse, Algiers, and San Fernando, in which magnitude equation is discovered or suspected, with the mean of Paris, Catania, Bordeaux, in which it does not appear.

Table V.

Collected Results for Magnitude Equation.

R.A.	Toulouse minus Mean of P.C.B.	San Fernando minus Mean of P.C.B.	Algiers minus Mean of P.C.B.
m	s	s	s
− 7·4	+0·001	−0·015	−0·031
7·5 − 8·4	0	− 10	− 6
	s	s	s
8·5 − 9·2	+ 1 − 0·001	− 2 − 0·005	+ 14 + 0·025
9·3 − 9·8	5 + 2	+ 10 − 14	+ 23 + 57
9·9 − 10·5	0	− 19	+ 69
10·6 − 11·2	− 7	− 23	+ 73
11·3 − 12·1	− 6	− 33	+ 79
12·2 −	...	...	+ 77
Decl.			
	"	"	"
− 7·4	+0·22	+0·14	+0·38
7·5 − 8·4	+ 5	− 4	+ 8
	"	"	"
8·5 − 9·2	0 − 0·18	− 6 − 0·16	− 2 − 0·16
9·3 − 9·8	− 15 − 23	− 18 − 17	− 16 − 24
9·9 − 10·5	− 32	− 24	− 28
10·6 − 11·2	− 37	− 25	− 38
11·3 − 12·1	− 48	− 22	− 44
12·2 −	...	...	− 35

The first of the two columns belonging to each observatory is derived from the *repère* stars, the second from the comparison stars. The roughness at the points where these overlap need not be taken too seriously, for the magnitudes of the first are Tucker's magnitudes, *i.e.* practically B.D. magnitudes, while those of the second are photographic magnitudes.

The conclusions, in brief, are as follows:—

Toulouse has very little magnitude equation in R.A., but a large one in declination, which is nearly linear.

Algiers has a very large magnitude equation in both co-ordinates; larger in R.A. than in Decl.; not strictly linear in either, but altering little for the faintest stars.

San Fernando appears to have a fairly small and decidedly non-linear magnitude equation in both co-ordinates.

In estimating the weight of these conclusions we must remember that there are other systematic errors involved with the magnitude equation, which have not yet been discussed; and that the actual quantity upon which magnitude equation must depend is likely to be the number representing the intensity of the image upon the plate, rather than the number more or less reduced to photometric scale which is derived from it and published as the photographic magnitude (except apparently at Paris). For these reasons no

attempt will be made just yet to determine a set of definitive corrections.

9. For the same reasons it is premature to attempt to settle the question—What is the cause of these errors? But we may note in passing, that at Toulouse and at Algiers the photographic equatorial is mounted in the "English" style,* so that errors of the objective are not reversed on crossing the meridian; while the plates are reversed during measurement. One will in this case naturally look first for the error in the objective. At San Fernando the plates are not reversed during measurement, and the small non-linear magnitude equation may very well be due to this cause.

10. *Progressive Discordances. Repère Stars.*—If a photographic telescope gives results which are systematically wrong over a long series, there is naturally a strong probability that the cause is instrumental; and if so, the effect will most likely be a function of the magnitude. We have therefore dealt with magnitude equation first.

We have now to look for discordances of a semi-systematic character, varying from point to point along the path of the planet, —such discordances, for example, as might be due to roughness in the adopted places of the *repère* stars. We will look for them first in the concluded photographic places of the *repère* stars themselves.

Each *repère* star has been assigned to one or other of a series of groups on centres 2° apart along the orbit of the planet (alternate centres of list given in Paris Circular No. 3, p. 7, for the "special series" plates).

11. Consider in the first place the observatories which have reduced to a common fundamental system (that of M. Loewy), viz. Catania, Greenwich, Paris, and Toulouse. The group means of the differences of these series are given in the table on the next page.

In studying this table we must remember—

(*a*) That the Greenwich plates were reduced first of all to an independent system, and afterwards mean corrections were applied to each plate to reduce to Loewy's system.

(*b*) That the Toulouse system is not exactly the same as Loewy's, though for present purposes the difference is insignificant.

(*c*) That the plates taken at Catania were measured and reduced at Paris, and ought to be particularly concordant with the Paris plates.

(*d*) That the Tolouse declinations are slightly affected by magnitude equation.

(*e*) That stars appearing in the same group are not necessarily reduced with the same selection of *repère* stars, or even with any single star common to the two reductions.

* I am indebted for this information to the directors of the observatories in question. It is not a little curious that, although the astrographic telescopes have now been at work for fifteen years, one cannot always find the slightest account of the type of mounting employed. Yet the question whether or not the objective is reversed on crossing the meridian is fundamental in all inquiries about systematic error.

(*f*) On the other hand, that the mean photographic place of one star will often depend on a considerable range of *repère* stars grouped on different centres, and that the errors of individual adopted places will be smoothed out much more effectually than is the case on Astrographic Catalogue plates, with their centres at uniform distances.

Bearing these facts in mind, we may draw the following conclusions from Table VI. :—

(1) The systematic difference between Paris and Greenwich, taken over the whole series, is quite insensible. And we found that there was no trace of relative magnitude equation between

Table VI.

Repère Stars reduced to Loewy's System. Progressive Discordances.

Centre.	Date. 1900.	Paris minus—Greenwich.	Toulouse.	Catania.
		s ″	s ″	s ″
6	Oct. 2	(11) ·000 + ″·03	(11) + ·005 + ″·15	(11) − ·001 + ″·02
8	,, 8	(15) + 3 − 2	(17) + 4 − 1	(17) + 3 0
10	,, 14	(11) − 13 + 4	(15) − 7 + 8	(13) + 22 + 20
12	,, 19	(15) 0 − 3	(20) 0 0	(14) − 3 + 3
14	,, 25	(11) + 1 0	(13) + 6 + 4	(12) + 26 + 6
16	Nov. 1	(14) + 13 + 1	(17) + 3 − 1	(16) + 11 + 10
18	,, 7	(16) − 9 0	(21) + 2 − 1	(21) − 6 + 6
20	,, 14	(16) + 3 − 1	(19) + 16 + 7	(13) + 2 + 1
22	,, 21	(16) − 7 + 4	(18) − 4 + 4	(12) − 8 + 18
24	,, 27	(15) + 3 − 1	(16) + 5 + 3	(13) + 17 + 21
26	Dec. 3	(7) + 4 − 21	(15) 0 + 2	(9) + 21 − 3
28	,, 9	(11) + 5 + 4	(11) + 2 0	(3) + 46 − 41
30	,, 14	(6) + 3 + 5	(7) + 15 − 20	(5) − 1 − 5
32	,, 18	(14) + 4 + 6	(14) + 4 + 3	(6) + 21 + 44
34	,, 22	(16) − 5 + 7	(16) + 1 + 3	...
36	,, 26	(16) − 4 + 2	(17) + 3 − 5	(1) − 18 + 40
Average without regard to sign		s ″ ·0048 ″·040	s ″ ·0048 ″·048	s ″ ·0137* ″·147*
Mean		·0000 + ·028	+ ·0033 + ·072	+ ·0060 + ·086

Note.—The averages are formed from the group means given above, and sometimes the same star occurs in two groups. These duplicates have been expunged before the final means were taken, in which no star is reckoned more than once.

them. We may provisionally adopt the figures in the columns Paris—Greenwich as indicative of the degree of concordance that

* Or, omitting the last four, which depend on few observations, the averages are s 0·0109 ″0·082.

may be reached by two series made independently with different instruments and reduced to the same standard.

(2) The divergences from centre to centre seem to be due more to the roughness of the *repère* star places than to errors of measurement, or real errors of the photographs. This may be seen in the columns for Paris—Toulouse. The Toulouse plates are few in number compared with Greenwich and Paris. But the average discordance of the groups is nearly the same in Paris—Toulouse as it is in Paris—Greenwich.

(3) The Catania plates are still fewer in number, being practically only alternate plates of the special series. Although they were measured and reduced at Paris, the discordances from Paris are large. At first sight this might be attributed to small number of plates and want of averaging out errors by overlapping. But the discordances seem to be too large for any such simple explanation; notice especially centres 10, 14, 24, and 26. We will postpone discussion of them till we come to deal with the fainter stars.

Table VII.

Repère Stars not reduced to Loewy's System. Progressive Discordance from Paris of Photographic and Adopted R.A.'s.

		Paris minus—							
		San Fernando.		Northfield.		Helsingfors.			
Centre.	Date.	Phot.	Adopt.	Phot.	Adopt.	Phot.	Adopt.		
	1900.	s	s	s	s	s	s		
6	Oct. 2	(11)	+·004	+·009	...	...	...	...	
8	,, 8	(17)	+13	+6	(13)	+·019	+·005	...	...
10	,, 14	(15)	+3	+9	(14)	−1	+8	...	...
12	,, 19	(20)	+10	+10	(15)	+12	+9	(8)	+·008 +·010
14	,, 25	(13)	+9	+11	(10)	+10	+5	(9)	+32 +10
16	Nov. 1	(17)	+11	+15	(11)		0+10	...	...
18	,, 7	(21)	+15	+15	(14)	−1	+11	(18)	−3 +9
20	,, 14	(19)	+21	+9	(11)	+3	+7	(4)	−13 +10
22	,, 21	(18)	+13	+11	(10)	−3	+6	(13)	+7 +11
24	,, 27	(16)	+11	+11	(14)	+16	+8	(15)	+22 +17
26	Dec. 3	(15)	−1	0	(11)	+13	+11	(13)	+25 +14
28	,, 9	(11)	+5	+8	(8)	+2	+8	(12)	+18 +13
30	,, 14	(7)	+5	+3	...	...		...	...
32	,, 18	(14)	+5	−1	(11)	+3	+7	...	...
34	,, 22	(16)	−3	+5	(14)	+4	+4	(11)	+9 +10
36	,, 26	(17)	+11	+3	(11)	+3	+6	(12)	+15 +8
	Mean		+·0082	+·0078		+·0057	+·0075		+·0120 +·0112

(4) Briefly, then, we conclude from Table VI. that it is possible to get consistent places, both R.A.'s and declinations, from two photographic series made at different observatories—witness Paris

and Greenwich ; but even under conditions which should make for accordance, one finds discordances unexpectedly big, as in the case of Catania.

12. So far we have dealt with series reduced to the same fundamental system. We must now examine how far difference of adopted system is likely to produce discordance.

We will take as examples San Fernando, Northfield, and Helsingfors. The comparison of R.A.'s, both deduced and adopted, is given in Table VII.

It will be seen at once that the difference in the adopted places is responsible for the principal part of the difference in the concluded photographic places. At San Fernando and Helsingfors the mean difference is exactly accounted for. At Northfield the agreement is less satisfactory, but the series is less complete.

13. The declinations have been treated in the same way, and give a very similar result. The mean discordances of the photographic declinations are due almost entirely to the difference of adopted places.

I think we may conclude that the policy of allowing any observatory to make its own system of adopted places for the *repère* stars has led to well-marked systematic discordances between some of the photographic series, of the order of $0^s.01$ and $0''.1$, which it will be worth while to eliminate.

14. *Comparison Stars. Progressive Discordances.*—We have now to examine how far the photographic places of the fainter stars differ from centre to centre. Our material is the same as that discussed above for magnitude equation. Each observatory has been compared with each of the others. The results have been studied by plotting in curves coloured to indicate the observatory of origin. These cannot be reproduced, and the tables which represent them numerically are not quite successful in showing up the features which are conspicuous in the diagrams. We can give only a few examples, and a general statement of results.

A typical section is that traversed by the planet between October 16 and November 4, which has been divided into blocks lettered K, L, M, N, O. Table VIII. gives the R.A. comparisons.

Since all these comparisons are differential, it is impossible to obtain the actual error of any series. But we sometimes can get a very good idea of which is in the wrong. Take, for instance, the comparison of Catania with others. In block K the Catania R.A.'s are about $0^s.04$ greater than all the others ; in block L they nearly agree ; in block M they are decidedly less ; in block N they have risen again ; and fall once more in O as compared with the others. I think that we may conclude in this case that the irregularity is mainly in the Catania observations.

In this way it has been found possible to obtain a general idea of the degree of systematic error in the photographic places, both R.A.'s and declinations, of the comparison stars. The result is by no means reassuring. Let us take the observatories in order.

Table VIII.

Comparison Stars. Photographic R.A. Progressive Discordances between Results from Different Observatories.

	Paris –	Bord. –	Cat. –	S. Fern. –	Toul. –
K. Oct. 16–19.					
Paris	...	(14) + ˢ·012	(29) + ˢ·040	(14) – ˢ·021	(12) + ˢ·001
Bord.	(14) – ·012	...	(15) + 34	(13) – 20	(12) – 7
Cat.	(29) – 40	(15) – 34	...	(12) – 54	(9) – 37
S. Fern.	(14) + 21	(13) + 20	(12) + 54	...	(13) + 21
Toul.	(12) – 1	(12) + 7	(9) + 37	(13) – 21	...
L. Oct. 19–23.					
Paris	...	(28) + ·009	(16) – ·002	(19) – ·003	(26) – ·003
Bord.	(28) – ·009	...	(10) – 5	(17) – 10	(24) – 12
Cat.	(16) + 2	(10) + 5	...	(6) – 13	(8) – 12
S. Fern.	(19) + 3	(17) + 10	(6) + 13	...	(20) – 4
Toul.	(26) + 3	(24) + 12	(8) + 12	(20) + 4	...
M. Oct. 23–27.					
Paris	...	(29) – ·002	(36) – ·030	(39) – ·008	(32) + ·011
Bord.	(29) + ·002	...	(36) – 22	(36) – 29	(23) + 15
Cat.	(36) + 30	(36) + 22	...	(37) + 3	(23) + 39
S. Fern.	(39) + 8	(36) + 29	(37) – 3	...	(28) + 28
Toul.	(32) – 11	(23) – 15	(23) – 39	(28) – 28	...
N. Oct. 27–31.					
Paris	...	(31) – ·003	(38) – ·003	(32) – ·033	(19) – ·023
Bord.	(31) + ·003	...	(35) + 10	(35) – 38	(18) – 5
Cat.	(38) + 3	(35) – 10	...	(34) – 41	(21) – 19
S. Fern.	(32) + 33	(35) + 38	(34) + 41	...	(20) + 32
Toul.	(19) + 23	(18) + 5	(21) + 19	(20) – 32	...
O. Nov. 1–4.					
Paris	...	(20) – ·016	(33) – ·026	(21) – ·002	(22) – ·031
Bord.	(20) + ·016	...	(28) – 13	(21) + 21	(21) – 2
Cat.	(33) + 26	(28) + 13	...	(22) + 30	(24) + 7
S. Fern.	(21) + 2	(21) – 21	(22) – 30	...	(22) – 21
Toul.	(22) + 31	(21) + 2	(24) – 7	(22) + 21	...

15. Paris comes out decidedly the best. Only in a few cases is Paris systematically different from all the others.

Bordeaux results are also good. The differences from other observatories are very irregular, and to be attributed in the main to the others, and not to Bordeaux.

Catania is an extremely interesting case. The discordances from Paris are quite extraordinary. The following table gives their values in the section where they are best determined:—

Table IX.

Comparison Stars. Progressive Discordances. Paris minus Catania.

Block.	No. of Stars.	Mean Discordance in R.A.	Decl.
		s	″
E.	(8)	−·013	−·06
F.	(32)	− 18	− 5
G.	(12)	+ 32	− 9
H.	(30)	− 21	+ 24
K.	(29)	− 40	+ 12
L.	(16)	− 2	− 12
M.	(36)	+ 30	+ 12
N.	(38)	+ 3	− 19
O.	(33)	+ 26	− 7
P.	(39)	+ 2	+ 7
Q.	(41)	− 26	− 13
R.	(38)	+ 21	0
S.	(4)	0	+ 13
T.	(11)	+ 34	− 11
U.	(11)	− 40	+ 11
V.	(2)	− 25	− 5
X.	(7)	+ 74	− 2

The greater part of the error is certainly in the Catania places. It is less in Decl. than in R.A. It does not correspond at all well with the rather large discordance between Paris and Catania for the *repère* stars, and cannot be due to roughness in the adopted places of the stars, for the places where it is greatest are generally those where Paris plates are most numerous.

Up to the present I have not succeeded in determining the form of this error, or in finding any explanation of it. That two series of plates, taken indeed with different telescopes, but measured and reduced by the same hands, should give results so remarkably discordant, could hardly have been expected by the most avowed disbeliever in the accuracy which is claimed, and generally with truth, for the photographic methods.

Toulouse and San Fernando exhibit great and very variable divergences from the others; but we have already shown that these series are affected by rather large magnitude equations, which in a climate of variable transparency must necessarily be associated with semi-systematic mean errors of the kind we are

now considering. The only sure method of procedure in such a case seems to be a discussion for magnitude equation plate by plate, which is impossible unless the published material is amplified.

16. *Conclusion.*—The results of this discussion make it clear that the star places published in the Paris Circulars are by no means homogeneous, and that errors exist in some of the series which, uneliminated, would ruin the determination of the solar parallax. I have, however, full confidence that these difficulties can be overcome. By the great kindness of the directors of several observatories, the separate results from each of their plates have recently been placed at my disposal, and the discussion of this material is now proceeding at Cambridge, with results which I hope to communicate to the Society.

17. But our conclusions have, it seems to me, an interest wider than that of the particular problem in hand. Most of the photographic telescopes concerned have been engaged for years upon the Astrographic Chart and Catalogue. With that experience behind them, they undertook shares in the Eros co-operation; and several of the resulting series of star places are affected by errors much larger than are accounted tolerable in the Astrographic Catalogue—a disquieting result.

18. The work summarised in this paper was done concurrently with that described in paper No. 4, and the same acknowledgments are due to the Government Grant Fund of the Royal Society; and to Miss Julia Bell, who has carried out the greater part of the computation.

Cambridge Observatory:
1906 *November* 7.

On the Distribution of Energy in the Continuous Spectrum. By E. T. Whittaker, Sc.D., F.R.S., Royal Astronomer of Ireland.

The distribution of energy in the continuous spectrum of the "black body" at various temperatures, which is of interest astronomically from its application to the problem of solar and stellar temperatures, has been studied from the experimental side by many physicists, and the results obtained have been co-ordinated into the form of empirical laws of radiation. One such empirical law, which is due to Lummer and Pringsheim,* and closely represents the observations, is to the effect that the intensity in the part of the spectrum at wave-length λ, radiated by a black body at the absolute temperature T, is proportional to

$$T\lambda^{-4}e^{-\frac{c}{(\lambda T)^{1\cdot 3}}} \quad . \quad . \quad . \quad . \quad . \quad (I.)$$

where c is a constant.

The mode of occurrence of T in this formula is not really empirical, as it can be proved by thermodynamical considerations

* *Deutsch. Phys. Gesells. Verh.*, 2, 12*a*, 103–180 (1900).

that if the energy between wave-lengths λ and $\lambda + d\lambda$, when the body is at temperature T_0, is $\psi(\lambda)d\lambda$, then the corresponding quantity when the body is at temperature T is

$$\left(\frac{T}{T_0}\right)^5 \psi\left(\frac{T\lambda}{T_0}\right)d\lambda \quad . \quad . \quad . \quad \text{(II.)}$$

Lummer and Pringsheim's formula is evidently constructed to satisfy this condition, and what is really empirical in the formula is therefore the mode of occurrence of λ when T is constant; in other words, the distribution of energy in the spectrum at some one definite temperature is taken from observation, and the distribution of energy in spectra at all other temperatures can then be deduced from thermodynamical principles.

For the above formula (I.) no theoretical justification has yet been found. But one feature, common to it and to all the rival formulæ which have been suggested since the results of observations in the extreme infra-red have been available, is that the distribution of energy in the region of long wave-lengths is proportional to

$$T\lambda^{-4}d\lambda \quad . \quad . \quad . \quad . \quad \text{(III.)}$$

In other words, the curve of intensity I in the spectrum must approximate to the curve $I = CT\lambda^{-4}$ in the ultra-red where C is a constant.

This result (III.) may now be regarded as a well-established result of observation;* and it becomes important to inquire whether it can be explained on theoretical grounds.

Several writers have discussed this question by the aid of assumptions regarding the nature of the radiating mechanism in the black body. Lord Rayleigh † suggested the application of the Boltzmann-Maxwell doctrine of the partition of energy among the different modes of vibration. The difficulty here lies in the doctrine itself, which is not free from uncertainties, and would give results inconsistent with observation if applied to the shorter wave-lengths.

Planck ‡ has attacked the matter from the point of view of a distinctive theory of the mechanism of radiation. The radiating body is supposed to contain a great many electrical vibrators, each having its own period of free vibration, and exchanging energy with the æther and the material molecules. By discussion of such a system, Planck derives the law of radiation

$$\frac{c_1\lambda^{-5}}{e^{\frac{c_2}{\lambda T}} - 1}d\lambda,$$

* It is further confirmed by the observations of Rubens and Kurlbaum, *Ann. d. Phys.*, iv. p. 649 (1901).

† "Remarks upon the Law of Complete Radiation," *Phil. Mag.*, xlix. 539 (1900). Lord Rayleigh's paper was written before the accuracy of the law had been experimentally verified, and therefore constitutes a successful prediction from theory. See also Jeans, *Phil. Mag.* (6) x. 97 (1905).

‡ Ueber das Gesetz der Energieverteilung im Normalspectrum, *Ann. d. Phys.*, iv. 553 (1901).

where c_1 and c_2 are constants. This formula evidently satisfies law (III.), and is indeed in good accord with observation for the shorter wave-lengths also.

Later, Lorentz* gave a different view of the mechanism of radiation: he explains the emission of a metal by means of the heat-motion of its free electrons, which are regarded as moving to and fro with a velocity of agitation increasing with the temperature, and frequently striking against the molecules. He shows that such a system would emit the longer radiations in accordance with formula (III.) above.

The object of the present paper is to show that formula (III.) can be established on theoretical grounds quite apart from any assumptions as to the mechanism of radiation; that in fact it is a necessary consequence of the laws of thermodynamics, together with the usual assumptions regarding the nature of the white light.

Natural radiation is now generally understood to consist of a succession of discrete disturbances or "pulses" in the æther, which are not co-ordinated as regards phase, and each of which consists of compensating positive and negative parts, so that the curve representing a pulse has the total area of those portions of it which are below the axis equal to the total area of those portions which are above the axis. By the agency of a prism or grating, a single pulse of this kind is drawn out into trains of periodic disturbances, the dispersive apparatus in fact performing a resolution of the pulse which corresponds to that furnished analytically by Fourier's integral; and it is this resolution which constitutes spectroscopy.

Suppose, then, that a pulse in æther is represented by $f(x-ct)$, where x denotes distance measured in the direction of propagation of the pulse, t denotes time, and c is the velocity of light in æther. We shall express the discrete character of the pulse by supposing that $f(x-ct)$ is zero except when $x-ct$ lies between the limits a and b.

Then Fourier's resolution can be written

$$f(x-ct)=\frac{1}{\pi}\int_0^\infty dn\int_a^b \cos\{n(x-ct-\mu)\}f(\mu)d\mu$$

So when the pulse is spectroscopically resolved, the elements with wave-lengths between λ and $\lambda+d\lambda$ in æther will be (writing $2\pi/\lambda$ for n, and also for convenience writing $a+y$ for μ)

$$\frac{2d\lambda}{\lambda^2}\int_0^{b-a}\cos\left\{\frac{2\pi}{\lambda}(x-ct-a-y)\right\}f(a+y)dy$$

Now throughout the range over which the integration is taken, y is small compared with λ if the wave-length is taken so far in the infra-red that λ is large compared with the extent of the pulse. We can therefore expand the cosine in ascending powers of $\frac{y}{\lambda}$ and

* "On the Emission and Absorption by Metals of Rays of Heat of great Wave-lengths," *Proceedings of the Amsterdam Academy of Sciences* (English edition), v. 666 (1902).

retain only the leading terms of the expansion. Retaining for the present the first two terms, the preceding expression becomes

$$\frac{2d\lambda}{\lambda^2}\cos\left\{\frac{2\pi}{\lambda}(x-ct-\alpha)\right\}\int_0^{b-a} f(a+y)dy$$

$$+\frac{2d\lambda}{\lambda^2}\frac{2\pi}{\lambda}\sin\left\{\frac{2\pi}{\lambda}(x-ct-\alpha)\right\}\int_0^{b-a} yf(a+y)dy$$

The first of these integrals vanishes in consequence of the condition that the pulse consists of compensating positive and negative parts; the second integral depends on the particular form of the pulse, but is independent of λ, and will in general have a finite value different from zero, which we shall denote by C. The spectroscopic element of the pulse with wave-lengths between λ and $\lambda+d\lambda$ is therefore

$$\frac{4\pi C d\lambda}{\lambda^3}\sin\left\{\frac{2\pi}{\lambda}(x-ct-\alpha)\right\}$$

Now if in any disturbance the spectroscopic element with wave-lengths between λ and $\lambda+d\lambda$ is

$$f(\lambda)\,.\,d\lambda\,.\,\sin\left\{\frac{2\pi}{\lambda}(x-ct-\epsilon)\right\},$$

it is known * that the energy radiated in this interval is proportional to $\lambda^2\{f(\lambda)\}^2 d\lambda$. So in the present case the energy radiated with wave-lengths between λ and $\lambda+d\lambda$ is proportional to $\lambda^{-4}d\lambda$. As the various pulses are supposed to be entirely unco-ordinated as regards phase, this law which holds for each of them individually will hold also for their aggregate; and therefore, in the radiation emitted by any body, the energy in the part of the spectrum between λ and $\lambda+d\lambda$ is proportional to $\lambda^{-4}d\lambda$ in the region of longer wave-lengths.

From this result, by an application of the thermodynamical theorem (II.) above, we immediately deduce the consequence that the radiation of a body at temperature T is, in the ultra-red, proportional to $T\lambda^{-4}d\lambda$.

Law (III.) is thus established as a direct consequence of the nature of white light, without reference to the mechanism of radiation in the radiating body.

On the Resolving Power of Spectroscopes. By E. T. Whittaker, Sc.D., F.R.S., Royal Astronomer of Ireland.

The resolving power of a spectroscope is, according to the usual definition, the value of $\frac{\lambda}{\delta\lambda}$, where $\delta\lambda$ is such that the luminous centre of the spectral line of wave-length $\lambda+\delta\lambda$ falls on the first minimum of intensity of the line of wave-length λ. It is a well-

* *Cf.* Lord Rayleigh, *Phil. Mag.*, xxvii. 460 (1889).

known theorem that if θ denotes the deviation of light of wave-length λ, so that $\frac{d\theta}{d\lambda}$ measures the dispersion caused by the spectroscope, and if a denotes the breadth of the beam of parallel light of the wave-length λ as it emerges from the dispersive apparatus, then the resolving power of the instrument is $a\,\frac{d\theta}{d\lambda}$.

The resolving power of a spectroscope can also be discussed in connection with the modern view of the nature of luminous disturbance as a succession of pulses. This has been done by several writers.* In the case of the grating, it is evident that a single incident pulse is broken up into as many separate pulses as there are spacings in the grating: these separate pulses, as they issue from the grating, are no longer in the same wave-front, and they will consequently follow each other at regular intervals to the focus of the observing telescope. In this way we can establish for the grating a theorem to the effect that the number of alternations, in the disturbance which is formed from a single original pulse, is equal to the resolving power of the instrument.

The case of dispersion by a prism presented greater difficulties, as it seemed somewhat mysterious that alternations should be formed at all in this case. But the discussions of Professors Schuster and Ames and Lord Rayleigh † make it evident that the difference between the wave-velocity and the group-velocity of luminous disturbance in the material of the prism will cause a single incident pulse to be spread out into an alternating disturbance; and the working out of this idea verifies the same theorem for prisms as has been stated above for gratings, namely, that the number of pulses into which a single incident pulse is spread out is equal to the resolving power of the instrument.

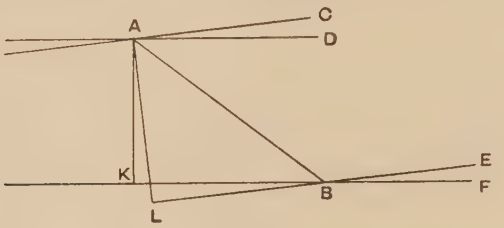
It becomes, therefore, of interest to determine whether this theorem can be established in the most general case by a proof which will be independent of the nature of the dispersive apparatus, and consequently applicable to every type of spectroscope. The object of the present note is to communicate such a proof.

Suppose, then, that a pulse, which for our present purpose we may regard as a thin plane sheet of disturbance moving perpendicularly to its own plane, is incident on any dispersive apparatus. For simplicity we shall suppose that the Fourier analysis of the pulse will yield radiations of wave-lengths λ and $\lambda + d\lambda$ only. At incidence the *pulse-front*, or plane on which the disturbance exists, is the same as the *phase-front*, or plane parallel to which the λ-disturbance is in the same phase; but after emergence from the dispersive apparatus this will no longer in general be the case. Suppose, then, that ϕ denotes the angle between the pulse-front and phase-front on emergence; and let $d\theta$ denote the angle between the

* *Cf. e.g.* Lord Rayleigh, Larmor, Schuster, and Ames.

† Schuster, *Theory of Optics*, p. 325 (1904); Ames, *Astrophysical Journal*, July 1905; Rayleigh, *Phil. Mag.*, x. p. 401 (1905).

phase-fronts for the λ radiation and the $\lambda+d\lambda$ radiation, so that $d\theta$ is the dispersion. If, now, A C, B E in the figure are two successive crests of the λ-disturbance at emergence, and A D, B F



two successive crests of the $\lambda+d\lambda$ disturbance, then the pulse-front will pass through the points A, B, where the crests reinforce each other. Thus if A K, A L are perpendicular respectively to B F and B E, we have

$$AK=\lambda, \qquad AL=\lambda+d\lambda, \qquad K\hat{A}L=d\theta, \qquad ABK=\phi$$

and so from the figure

$$AB\sin\phi=\lambda, \qquad AB\sin(\phi+d\theta)=\lambda+d\lambda$$

Subtracting these two equations, we have

$$AB\cos\phi\,d\theta=d\lambda$$

and dividing this into the first of them, we have

$$\tan\phi=\lambda\frac{d\theta}{d\lambda}$$

This equation, which is applicable to any dispersive apparatus, shows that the tangent of the angle between the pulse-front and the phase-front at emergence is equal to the product of the wave-length into the dispersion.

Since the resolving power is the product of the dispersion into the breadth of the emergent beam, it follows from this equation that the resolving power is equal to the product of $\frac{1}{\lambda}\tan\phi$ into the breadth of the emergent beam, *i.e.* it is equal to the product of $\frac{1}{\lambda}\sin\phi$ into the breadth of the emergent pulse. But $\frac{\lambda}{\sin\phi}$ is evidently A B in the figure, *i.e.* it is the interval between two successive maxima of intensity in the pulse-front; and thus the resolving power is equal to the quotient of the breadth of the emergent pulse by the interval between two successive maxima in it; that is, the resolving power is equal to the number of separate maxima of disturbance in the emergent pulse-front; which was to be proved.

Extended Nebula near 26 Ceti. By Dr Max Wolf, Assoc. R.A.S.

On four plates taken with the Bruce 16-inch telescope in September I find an extended nebula, which attracts attention both by its situation and its appearance.

Practically all extended nebulosities are found in or near the Milky-way, but this object is about 70° distant from the plane of the Milky-way. Its densest part is situated at

$$\alpha = 0^h\ 57^m{\cdot}4 \qquad \delta = +1^\circ\ 20' \qquad (1885{\cdot}0)$$

Round this is spread faint nebulous matter of varying intensity. At several points small clouds of greater density shine forth. Further out, the intensity is so feeble that it is impossible to exactly define its limits. It seems that these will be materially altered by lengthening the exposure. My exposures extend to four hours. With these the nebulous cloud extends about 40′ in declination and about 30′ in R.A. The three B.D. stars

	h m	
+1°.191	$\alpha = 0$ 56·4	$\delta = +1^\circ 24'$
1 .200	57·0	1 27
1 .201	57·3	1 32

are all involved in the cloud.

Southwards it reaches nearly to the B.D. star

$$+0^\circ{\cdot}207 \qquad 0^h\ 56^m{\cdot}9 \qquad +1^\circ 0'$$

but it cannot be certainly detected up to the star. Especially towards the west the nebulous matter promises to go much further.

Under the microscope the brighter parts of the mass are filled with numerous very small spots and short trails, so that the appearance is very different from that presented by the Milky-way nebulæ. It seems possible that this object is a multitude of very small planetary nebulæ collected in a cluster, which a more powerful instrument than mine may perhaps resolve.

Astrophysical Observatory, Heidelberg:
1906 *October.*

Observations of the Satellite of Neptune from Photographs taken at the Royal Observatory, Greenwich, between 1905 *December* 19 *and* 1906 *April* 25.

(*Communicated by the Astronomer Royal.*)

The following measures of position-angle and distance of Neptune's satellite were made from photographs taken with the 26-inch refractor of the Thompson equatorial. The occulting shutter was used as in previous years. The photographs were

taken by Messrs Davidson, Edney, or Melotte, and were measured in a position micrometer in direct and reversed positions by Messrs Davidson and Edney. The tabular positions with which comparison is made were computed from the data given in the *Connaissance des Temps*, based on Dr Hermann Struve's elements, the eccentricity of the orbit being neglected.

A discussion of these residuals gives the following differences in the sense Tabular–Observed to Dr Hermann Struve's elements

$$du = -0^\circ\cdot87 \quad dN = -1^\circ\cdot16 \quad dI = +0^\circ\cdot17 \quad da = +''\cdot090$$

giving for the epoch 1906·2

$$a = 16''\cdot181 \quad N = 188^\circ\cdot71 \quad I = 116^\circ\cdot51.$$

NEPTUNE AND SATELLITE.

Position-angle and Distance from Photographs taken with the 26-inch Refractor.

Date and G.M.T.					Position-angle.			Distance.		
					Observed.	Tabular.	T.–O.	Observed.	Tabular.	T.–O.
1905	d	h	m	s	°	°	°	″	″	″
Dec.	19	10	58	6	115·48	115·58	+0·10	15·02	15·46	+0·44
1906										
Jan.	13	11	7	27(*a*)	(45·38)	45·03	(−0·35)	(13·32)	12·93	(−0·39)
	13	11	35	28	42·43	43·68	+1·25	12·46	12·81	+0·35
	19	12	13	11(*b*)	32·04	32·62	+0·58	11·80	12·00	+0·20
	19	12	26	45	29·73	30·75	+1·02	12·01	11·89	−0·12
	22	11	8	9	212·11	211·36	−0·75	12·14	11·92	−0·22
	22	11	34	27(*c*)	208·37	209·88	+1·51	11·56	11·84	+0·28
Feb.	3	7	59	26	200·43	201·90	+1·47	11·63	11·43	−0·20
	3	8	30	0	199·21	200·05	+0·84	11·25	11·37	+0·12
	3	9	0	31	198·52	198·18	−0·34	11·09	11·30	+0·21
	3	9	29	57	195·08	196·34	+1·26	11·11	11·24	+0·13
	3	9	54	56	193·08	194·78	+1·70	11·14	11·20	+0·06
	12	9	44	45	359·23	358·54	−0·69	10·88	11·00	+0·12
	12	10	9	58(*g*)	356·21	356·90	+0·69	10·95	11·01	+0·06
	12	11	7	42	352·68	353·18	+0·50	10·83	11·05	+0·22
	14	10	2	25	245·45	246·53	+1·08	15·03	15·09	+0·06
	14	10	27	31	244·40	245·66	+1·26	14·98	15·00	+0·02
	14	10	54	48	243·99	244·69	+0·70	14·79	14·88	+0·09
	14	11	16	4(*b*)	242·63	243·94	+1·31	14·82	14·80	−0·02
	15	9	13	7(*d*)	174·71	174·91	+0·20	11·06	11·02	−0·04
	20	9	44	29	239·21	240·83	+1·62	14·41	14·44	+0·03
	20	10	11	26	238·34	239·81	+1·47	14·32	14·33	+0·01
	20	11	14	39(*f*)	236·88	237·38	+0·50	14·22	14·07	−0·15
	23	10	36	42(*h*)	54·10	55·38	+1·28	13·47	13·84	+0·37
	23	11	8	31(*g*)	53·25	54·08	+0·83	13·19	13·76	+0·57
Mar.	2	9	35	20(*e*)	326·92*	327·27	+0·35*	11·38*	11·99	+0·61*

* Half weight.

NEPTUNE AND SATELLITE—*continued.*

Date and G.M.T.				Position-angle.			Distance.		
				Observed.	Tabular.	T.–O.	Observed.	Tabular.	T.–O.
1906	d	h	m s	°	°	°	″	″	″
	2	10	2 6(*c*)	324·13	325·85	+1·72	12·05	12·09	+0·04
	2	10	31 44	324·89	324·30	−0·59	12·01	12·20	+0·19
	2	10	54 35	322·23	323·13	+0·90	12·10	12·28	+0·18
	3	10	13 17	272·44	273·23	+0·79	16·32	16·57	+0·25
	3	10	46 39	271·03	272·31	+1·28	16·50	16·59	+0·09
	3	11	15 41(*c*)	269·96	271·50	+1·54	16·29	16·59	+0·30
	7	9	50 59(*c*)	42·14	41·69	−0·45	12·73	12·49	−0·24
	7	10	18 28(*c*)	39·38	40·33	+0·95	12·27	12·38	+0·11
	7	10	44 20(*h*)	(34·29)	39·03	(+4·74)	(12·54)	12·28	(−0·26)
	7	11	13 13	36·85	37·55	+0·70	12·11	12·17	+0·06
	12	8	37 32	87·88	88·49	+0·61	16·84	16·50	−0·34
	12	9	6 27	86·84	87·68	+0·84	16·65	16·48	−0·17
	12	9	47 24	85·73	86·53	+0·80	16·97	16·45	−0·52
	22	10	25 50	192·30	194·89	+2·59	11·03	10·96	−0·07
	22	10	50 41	192·73	193·32	+0·59	11·03	10·92	−0·11
	29	9	20 53(*j*)	113·24	114·16	+0·92	14·71	14·88	+0·17
	29	9	50 37(*f*)	111·06	113·17	+2·11	15·16	14·99	−0·17
Apr.	3	8	38 41(*k*)	178·90	179·05	+0·15	11·10	10·70	−0·40
	3	9	5 39(*d*)	176·31*	177·32	+1·01*	10·77*	10·71	−0·06*
	3	9	36 35(*c*)	174·96	175·31	+0·35	10·97	10·73	−0·24
	3	9	59 32(*j*)	(169·80)	173·84	(+4·04)	(11·50)	10·75	(−0·75)
	4	8	8 17	109·36	110·75	+1·39	15·11	15·18	+0·07
	4	8	41 8	108·17	109·70	+1·53	15·21	15·28	+0·07
	4	9	7 7	106·93	108·88	+1·95	15·22	15·36	+0·14
	4	9	31 40	107·38	108·11	+0·73	15·24	15·43	+0·19
	6	8	2 7(*c*)	355·38	355·69	+0·31	10·42	10·70	+0·28
	6	8	31 33	352·96	353·80	+0·84	10·75	10·73	−0·02
	6	9	4 38	352·31	351·68	−0·81	10·72	10·76	+0·04
	6	9	36 11	351·09	349·67	−1·42	10·75	10·80	+0·05
	9	8	2 57(*h*)	169·98	169·96	−0·02	10·67	10·77	+0·10
	9	8	38 56	165·97	167·68	+1·71	10·97	10·82	−0·15
	9	9	8 12(*e*)	165·54*	165·86	+0·32*	10·99*	10·87	−0·12*
	10	8	13 36	103·34	105·12	+1·78	15·50	15·64	+0·14
	10	8	40 55(*j*)	103·50	104·01	+0·51	15·53	15·68	+0·15
	14	9	21 33	234·79	235·96	+1·17	13·79	13·52	−0·27
	19	8	49 48	275·68	276·44	+0·76	16·09	16·07	−0·02
	25	8	53 59(*f*)	269·24	271·41	+2·17	16·01	16·11	+0·10

(*a*) Image of Satellite imperfect.
(*b*) Neptune near edge of shutter.
(*c*) Diffused.
(*d*) Very diffused.
(*e*) Extremely diffused.
(*f*) Faint.
(*g*) Faint and diffused.
(*h*) Very faint and diffused.
(*j*) Extremely faint.
(*k*) Satellite touching fringe of secondary spectrum.

* Half weight.

Royal Observatory, Greenwich:
1906 *November* 7.

Observations of Comet c 1905, *from Photographs taken with the 30-inch Reflector of the Thompson Equatorial.*

(*Communicated by the Astronomer Royal.*)

The following positions of Comet *c* 1905 were obtained from photographs taken with the 30-inch reflector.

Exposures of 4^m and 5^m were given on December 8 and 9, from 1^m to 3^m on December 19, and $\frac{1}{2}^m$ for the remainder.

The plates were measured in the astrographic micrometer. Six reference stars were taken where possible, situated as symmetrically as possible about the comet. The positions of the stars were taken from the catalogues of the Astronomische Gesellschaft, or from Karlsruhe Observations 1885·0, or the Radcliffe Catalogue 1890.

Date and G.M.T. 1905.				Apparent R.A.			Apparent Decl.			Log Δ.	Corr. for Parallax. R.A.	Corr. for Parallax. Decl.
d	h	m	s	h	m	s	°	′	″		s	″
Dec. 8	17	12	41	14	31	18·99	+20	5	4·6	0·1475	−0·25	+4·0
9	17	8	3	14	36	10·66	+19	36	56·7	0·1409	−0·25	+4·1
9	18	7	51	14	36	23·48	+19	35	45·6	0·1409	−0·21	+3·8
19	17	30	37	15	30	26·34	+13	46	40·0	0·0903	−0·27	+4·9
19	17	58	6	15	30	33·11	+13	45	52·8	0·0903	−0·25	+4·8
1906.												
Jan. 6	18	40	17	17	34	40·56	−	3	10	49·9	0·0425	−0·28	+6·3
7	18	46	53	17	42	39·12	−	4	21	40·3	0·0425	−0·28	+6·4
9	18	46	6	17	58	56·01	−	6	46	9·6	0·0441	−0·29	+6·4
10	18	39	27	18	7	16·25	−	7	59	46·0	0·0457	−0·29	+6·4

Royal Observatory, Greenwich:
1906 *November* 9.

Observations of Comet a 1906, *from Photographs taken with the 30-inch Reflector of the Thompson Equatorial.*

(*Communicated by the Astronomer Royal.*)

The following positions of Comet *a* 1906 were obtained from photographs taken with the 30-inch reflector, with exposures of from 3 minutes on January 30 to 20 minutes on April 14.

The plates were measured in the astrographic micrometer, and the position of the comet is deduced from the means of six reference stars whose places were derived from the catalogues of the Astronomische Gesellschaft except on February 20, when Greenwich Observations for the catalogue for 1900·0 were used.

Date and G.M.T. 1906.				Apparent R.A.			Apparent Decl.			Log Δ.	Corr. for Parallax. R.A.	Corr. for Parallax. Decl.
	d	h	m s	h	m	s	°	′	″		s	″
Jan.	30	15	51 44	16	16	11·57	+53	33	2·3	9·9992	−0·52	+1·7
Feb.	5	9	34 14	16	4	27·17	64	26	20·7	9·9738	−0·55	+7·3
	7	12	21 16	15	55	42·81	68	44	52·1	9·9688	−1·08	+3·3
	7	12	33 59	15	55	39·88	68	45	55·9	9·9688	−1·08	+3·0
	12	8	54 41	15	5	55·23	78	39	7·4	9·9658	−1·52	+5·3
	20	11	59 4	7	22	4·42	81	18	18·6	9·9912	+1·57	−3·2
	23	9	53 13	6	27	35·17	76	25	27·1	0·0087	+0·63	−3·2
Mar.	2	11	42 46	5	50	22·43	65	15	23·4	0·0589	+0·71	+0·9
	3	11	54 51	5	48	20·29	63	50	44·0	0·0669	+0·68	+1·3
	3	12	5 54	5	48	19·32	63	50	6·5	0·0669	+0·69	+1·5
	5	10	26 39	5	45	21·95	61	16	16·6	0·0828	+0·50	+0·3
	5	10	47 26	5	45	20·91	61	15	10·5	0·0828	+0·54	+0·6
	6	9	22 21	5	44	16·01	60	3	59·8	0·0909	+0·37	−0·3
	6	9	34 49	5	44	15·31	60	3	21·0	0·0909	+0·39	−0·1
	22	11	33 10	5	41	35·36	45	3	57·7	0·2135	+0·32	+2·9
	28	8	6 18	5	43	51·45	41	26	8·2	0·2567	+0·18	+1·3
	29	8	23 28	5	44	19·54	40	52	35·6	0·2634	+0·19	+1·5
	29	8	43 4	5	44	20·15	40	52	10·0	0·2634	+0·21	+1·6
Apr.	14	8	33 51	5	53	38·51	+33	59	21·1	0·3602	+0·17	+1·9

Royal Observatory, Greenwich:
1906 *November* 9.

Observations of Comet b 1906, *from Photographs taken with the 30-inch Reflector of the Thompson Equatorial.*

(*Communicated by the Astronomer Royal.*)

The following positions of Comet *b* 1906 were obtained from photographs taken with the 30-inch reflector, with exposures of from 5 minutes to 10 minutes.

The plates were measured in the astrographic micrometer.

The position of the comet is referred to the mean of six reference stars whose places were derived from the catalogues of the Astronomische Gesellschaft.

Date and G.M.T. 1906.				Apparent R.A.	Apparent Decl.	Log Δ.	Corr. for Parallax. R.A.	Corr. for Parallax. Decl.
	d	h m	s	h m s	° ′ ″		s	″
Mar.	6	10 26	11	11 34 43·86	+1 43 34·2	9·6760	−0·43	+14·2
	6	10 39	49	11 34 43·60	+1 43 34·9	9·6760	−0·39	+14·2
	7	11 46	41	11 34 16·60	+1 45 5·1	9·6850	−0·16	+13·8
	7	11 56	1	11 34 16·32	+1 45 5·4	9·6850	−0·13	+13·8
	22	12 30	1	11 27 51·41	+2 7 36·4	0·3777	+0·04	+ 2·8
	29	11 2	13	11 25 13·63	+2 16 47·7	0·3861	0·00	+ 2·7
Apr.	3	10 30	21	11 23 35·88	+2 22 10·9	0·3935	−0·01	+ 2·7
	3	10 44	27	11 23 35·70	+2 22 10·7	0·3935	0·00	+ 2·7
	4	10 15	46	11 23 18·33	+2 23 6·0	0·3951	−0·01	+ 2·7
	4	10 29	25	11 23 18·17	+2 23 6·5	0·3951	0·00	+ 2·7
	9	10 15	22	11 21 59·65	+2 26 51·8	0·4035	0·00	+ 2·6
	9	10 25	21	11 21 59·48	+2 26 53·6	0·4035	+0·01	+ 2·6
	10	9 5	13	11 21 46·70	+2 27 22·8	0·4053	−0·04	+ 2·6
	14	10 5	30	11 20 59·65	+2 29 2·4	0·4128	+0·01	+ 2·6
	19	10 2	19	11 20 19·44	+2 29 29·1	0·4229	+0·02	+ 2·5
	20	9 48	56	11 20 13·93	+2 29 21·0	0·4250	+0·01	+ 2·5
	25	9 53	27	11 19 59·15	+2 27 29·9	0·4356	+0·03	+ 2·4
	26	9 52	27	11 19 58·79	+2 26 53·1	0·4378	+0·03	+ 2·4
	27	9 47	3	11 19 59·34	+2 26 12·3	0·4401	+0·03	+ 2·4
	28	10 15	16	11 20 0·87	+2 25 25·2	0·4423	+0·04	+ 2·4

Royal Observatory, Greenwich:
1906 *November* 9.

Continued from Vol. LXIV. p. 783.

Comet a 1904, *Brooks = Comet* 1904 *I.*

Greenwich Mean Time of Observation.				♂ − ✳ R.A.	No. of Comp.	Apparent R.A. of Comet.			♂ − ✳ Decl.	No. of Comp.	App. Decl. of Comet.			Log. Factor of Parallax in α.	in δ.	Star of Comp.
1904		h	m	s			h.	m	s													
Apr.	17	10	26	8·0	− 3 25·73	10	16	56	26·89	− 3	4″·8	5	+44	42	59″·7	−9·6782	+0·6068	1
	19	10	22	17·8	− 3 19·46	10	16	50	49·17	− 2	46·7	5	+45	57	45·1	−9·6823	+0·5753	2
	24	10	29	30·2	− 1 15·17	12	16	35	2·70	+ 5	20·0	4	+48	54	52·9	−9·6729	+0·4404	3
	24	10	29	30·2	− 0 54·31	12	16	35	2·57	− 8	8·6	4	+48	54	52·5	−9·6729	+0·4404	4
	30	10	3	30·1	− 46·41	10	16	13	6·52	+11	18·1	5	+52	1	36·4	−9·6798	+0·3768	5
May	2	9	37	11·0	+ 2 59·71	8	16	5	8·41	− 3	1·8	4	+52	56	10·4	−9·7010	+0·3269	6
	3	9	14	40·7	+ 10·65	10	16	1	3·75	− 6	57·0	5	+53	21	48·0	−9·6202	+0·3623	7
	5	9	40	16·3	− 32·66	10	15	52	29·40	− 7	20·8	5	+54	11	11·2	−9·6887	+0·1872	8
	5	9	40	16·3	+ 1 49·36	10	15	52	29·17	+ 1	37·1	5	+54	11	12·3	−9·6887	+0·1872	9
	7	10	1	31·8	+ 3 23·39	12	15	43	38·84	+ 5	35·1	4	+54	55	55·9	−9·6216	+9·9586	10
	9	9	59	20·2	+ 3 39·69	8	15	34	39·03	− 1	29·9	4	+55	35	46·7	−9·5975	+9·8024	11
	9	9	59	20·2	− 3 38·88	8	15	34	38·82	+ 1	19·7	4	+55	35	47·0	−9·5975	+9·8024	12
	14	10	50	2·3	+ 7 9·17	10	15	12	21·62	− 8	46·9	5	+56	55	30·3	−9·2589	−9·5884	13
	15	9	54	0·4	+ 1 36·80	8	15	6	49·23	+ 3	10·7	4	+57	7	27·9	−9·4829	−8·8546	13
	18	9	59	4·1	+ 1 16·28	10	14	52	48·17	+ 4	7·6	5	+57	37	35·0	−9·3573	−9·5961	14
	18	9	59	4·1	− 1 20·56	10	14	52	48·31	+11	41·9	5	+57	37	34·3	−9·3573	−9·5961	15
	19	9	36	22·5	+ 0·98	12	14	48	13·50	− 3	2·3	4	+57	45	19·2	−9·4252	−9·4944	16

Comet 1904 *I.*—(*continued*).

Greenwich Mean Time of Observation.		♂ − * R.A.	No. of Comp.	Apparent R.A. of Comet.	♂ − * Decl.	No. of Comp.	App. Decl. of Comet.	Log. Factor of Parallax in α.	in δ.	Star of Com
1904	h m s	m s		h m s	′ ″		° ′ ″			
May 20	9 52 11·2	− 1 40·36	12	14 43 34·11	+ 7 32·9	4	+57 52 8·4	−9·3041	−9·6985	17
24	9 40 16·0	+ 3 13·47	Ret.	14 25 31·80	+ 4 51·6	Ret.	+58 8 54·7	−9·1520	−9·8133	18
25	10 35 13·9	+ 1 21·09	12	14 20 58·62	+ 3 53·6	4	+58 10 44·7	+8·6157	−9·9565	19
25	10 35 13·9	− 4 49·66	12	14 20 58·22	− 15·7	4	+58 10 45·2	+8·6157	−9·9565	20
28	10 22 11·0	+13 26·09	Ret.	14 8 15·90	− 1 7·3	Ret.	+58 10 40·9	+8·8711	−9·8589	21
June 3	10 39 47·2	+ 52·07	12	13 44 54·06	+ 5 27·1	4	+57 49 45·0	+9·3941	−9·3404	22
4	10 55 30·1	+ 4 25·02	12	13 41 17·60	+ 2 23·3	4	+57 44 4·4	+9·5223	−8·8915	23
4	10 55 30·1	− 2 44·52	12	13 41 17·47	− 13·0	4	+57 44 4·9	+9·5223	−8·8915	22
5	10 50 57·0	+ 56·75	10	13 37 49·33	− 3 57·0	5	+57 37 44·1	+9·4937	−9·1129	23
12	10 40 15·8	+ 9·38	Ret.	13 16 3·93	+13 35·9	Ret.	+56 40 52·4	+9·5971	+9·6205	24
21	10 30 19·9	− 1 0·53	Ret.	12 54 29·20	− 5 52·1	Ret.	+55 6 0·3	+9·6721	+0·1008	25
28	10 40 35·2	+ 1 2·37	10	12 42 3·99	+ 7 2·8	5	+53 44 29·1	+9·7214	+0·3488	26
29	10 25 4·0	− 26·92	10	12 40 34·67	− 4 36·6	5	+53 32 49·8	+9·7088	+0·3196	26
July 7	11 16 10·2	− 1 12·26	10	12 30 34·59	− 4 34·9	5	+51 58 14·0	+9·7520	+0·5886	27
8	11 3 58·4	+ 35·26	10	12 29 34·67	+ 19·0	5	+51 46 45·1	+9·7486	+0·5771	28
8	11 3 58·4	+ 23·64	10	12 29 34·50	+ 14·4	5	+51 46 46·2	+9·7486	+0·5771	29
9	11 4 0·3	+ 0·21	12	12 28 36·53	− 3 16·6	4	+51 35 8·9	+9·7486	+0·5916	30
9	11 4 0·3	+ 2 26·30	12	12 28 36·41	+ 4 18·3	4	+51 35 9·3	+9·7486	+0·5916	31
11	11 3 36·0	+ 4 58·53	10	12 26 49·91	− 1 44·3	5	+51 12 13·3	+9·7465	+0·6115	32
16	10 40 16·2	+ 31·48	10	12 23 5·92	− 7 14·7	5	+50 16 30·2	+9·7418	+0·6108	33
18	10 59 32·9	+ 7·37	Ret.	12 21 51·87	− 6 35·7	Ret.	+49 54 39·8	+9·7355	+0·6737	34

Encke's Comet.

Greenwich Mean Time of Observation.				☄ − ✱ R.A.	No. of Comp.	Apparent R.A. of Comet.			☄ − ✱ Decl.	No. of Comp.	App. Decl. of Comet.			Log. Factor of Parallax in α.	Log. Factor of Parallax in δ.	Sta of Comp.
1904	h	m	s	m s		h	m	s	′ ″		°	′	″			
Nov. 26	6	44	11·0	+ 2 24·25	Ret.	21	30	51·33	+ 7 7·2	Ret.	+12	18	29·4	+9·1571	+0·7669	35
Dec. 5	6	53	18·2	+ 38·31	10	20	57	46·42	− 5 14·6	5	+ 7	4	19·5	+9·3820	+0·8134	36
5	6	53	18·2	+ 1 8·55	10	20	57	46·60	+ 3 16·9	5	+ 7	4	22·2	+9·3820	+0·8134	37
6	6	32	18·5	− 1 9·60	12	20	54	7·02	+ 2 10·9	4	+ 6	28	27·4	+9·3525	+0·8151	38
8	6	10	40·9	+ 1 22·82	12	20	46	37·15	+ 3 10·5	4	+ 5	14	46·6	+9·3119	+0·8205	39
8	6	10	40·9	+ 1 32·16	12	20	46	36·94	+ 7 49·4	4	+ 5	14	47·1	+9·3119	+0·8205	40
10	6	27	18·0	− 5·00	Ret.	20	38	49·57	2 13·1	Ret.	+ 3	57	40·7	+9·4040	+0·8303	41
13	6	0	35·3	− 1 4·50	10	20	26	43·85	− 4 56·4	5	+ 1	57	29·2	+9·3980	+0·8391	42
14	5	27	11·4	− 1 22·82	10	20	22	25·30	+ 3 40·9	5	+ 1	16	19·4	+9·3452	+0·8415	43
17	5	16	40·0	+ 2 51·96	10	20	9	14·62	+ 9 42·9	5	− 0	56	23·0	+9·3766	+0·8510	44
17	5	16	40·0	+ 14·66	10	20	9	14·60	+ 55·3	5	− 0	56	24·9	+9·3766	+0·8510	45

Comet e 1904 (*Borrelly*) = *Comet* 1905 *II.*

1905																
Jan. 4	6	27	30·0	− 7·71	Ret.	1	21	24·40	− 47·6	Ret.	− 5	10	46·9	+7·1072	+0·8731	46
8	6	29	49·8	− 3 44·87	10	1	27	16·12	− 3 3·0	5	− 1	56	40·5	+8·3042	+0·8578	47
9	6	42	39·1	− 17·42	10	1	28	49·16	− 2 44·3	5	− 1	8	6·4	+8·6384	+0·8535	48
9	6	42	39·1	+ 10·99	10	1	28	49·11	+ 7 40·8	5	− 1	8	9·3	+8·6384	+0·8535	49
11	7	22	40·3	+ 2 6·08	Ret.	1	32	0·60	+ 53·8	Ret.	+ 0	28	48·2	+9·0415	+0·8447	50
26	7	40	16·0	+ 1 3·20	Ret.	1	59	3·37	− 2 52·5	Ret.	+11	50	25·1	+9·2625	+0·7750	51

Comet a 1905 (*Giacobini*) = *Comet* 1905 *III.*

Greenwich Mean Time of Observation.		☄ − ✶ R.A.	No. of Comp.	Apparent R.A. of Comet.	☄ − ✶ Decl.	No. of Comp.	App. Decl. of Comet,	Log. Factor of Parallax in *a*.	in *δ*.	Star of Comp.
1905	h m s	m s		h m s	′ ″		° ′ ″			
Mar. 30	9 4 32·1	− 30·92	Ret.	5 58 43·36	+ 5 3·6	Ret.	+15 54 11·1	+9·4520	+0·7677	52
Apr. 3	9 15 11·9	+ 40·20	...	6 14 17·70	+ 8 0·5	...	+20 44 56·0	+9·4800	+0·7384	53
8	9 14 0·4	− 1 25·20	...	6 35 32·70	+ 3 8·0	...	+26 33 40·2	+9·4957	+0·6880	54

Comet b 1905 (*Schaer*) = *Comet* 1905 *V.*

Nov. 20	10 2 35·0	− 16·84	Ret.	23 54 42·79	−17 17·8	Ret.	+62 10 59·8	+9·5901	+0·3411	55
23	10 2 4·9	− 47·87	8	23 38 17·92	+ 5 59·2	4	+37 29 45·3	+9·4356	+0·4862	56
24	11 14 45·1	+ 49·38	10	23 35 53·63	− 4 1·8	5	+30 31 42·7	+9·5383	+0·6656	57
29	9 12 52·0	− 3 58·11	8	23 31 1·84	− 9 51·0	4	+ 9 23 8·0	+9·2930	+0·7972	58
30	6 11 32·8	− 1 52·46	Ret.	23 30 43·19	− 4 13·1	Ret.	+ 7 1 59·2	−8·8229	+0·8027	59

Comet b 1906 (*Kopff*) = *Comet* 1905 *IV.*

1906 Mar. 6	10 2 16·0	− 2 32·18	10	11 34 44·57	− 2 30·5	5	+ 1 43 32·8	−9·3744	+0·8398	60
8	9 43 28·9	− 3 23·27	10	11 33 53·51	+ 24·2	5	+ 1 46 27·3	−9·3945	+0·8399	60
8	9 43 28·9	+ 5 39·07	10	11 33 53·11	− 1 45·0	5	+ 1 46 26·2	−9·3945	+0·8399	61
15	9 27 11·3	+ 3 2·01	8	11 30 50·94	+ 5 30·8	4	+ 1 56 59·3	−9·3656	+0·8386	62
18	8 44 9·4	− 22·08	10	11 29 35·01	+ 40·3	5	+ 2 1 29·3	−9·4257	+0·8394	63
20	10 22 0·5	− 1 14·11	10	11 28 42·99	+ 3 45·7	5	+ 2 4 34·7	−9·1218	+0·8358	63
Apr. 18	9 31 5·0	− 34·93	8	11 20 25·61	+ 6 24·4	4	+ 2 29 39·3	−8·4235	+0·8324	64
20	9 26 53·1	+ 17·99	8	11 20 14·52	− 7 15·3	4	+ 2 29 22·1	−8·3151	+0·8325	65
28	9 16 24·2	+ 24·06	10	11 20 1·22	− 30·8	5	+ 2 25 25·9	+8·0789	+0·8329	66

Date. 1904	Star's Designation or Authority for Place.	Mean R.A. Equinox of Year. h m s	Corrections to Mean R.A. s	Mean Declination Equinox of Year. ° ′ ″	Correction to Mean Declination. ″	No. of Reference.
Apr. 17	A.G.Z. Bonn, No. 10909	16 59 51·06	+1·56	+44 46 11·9	− 7·4	1
19	,, ,, No. 10841	16 54 7·01	+1·62	46 0 39·0	− 7·2	2
24	,, ,, No. 10664	16 36 16·05	+1·82	48 49 38·6	− 5·7	3
24	,, ,, No. 10656	16 35 55·16	+1·72	49 3 6·1	− 5·0	4
30	A.G.Z. Harvard, No. 4954	16 13 50·87	+2·06	51 50 21·9	− 3·6	5
May 2	,, ,, No. 4911	16 2 6·59	+2·11	52 59 14·9	− 2·7	6
3	,, ,, No. 4903	16 0 50·96	+2·14	53 28 47·4	− 2·4	7
5	,, ,, No. 4869	15 52 59·88	+2·18	54 18 33·6	− 1·6	8
5	,, ,, No. 4862	15 50 37·63	+2·18	54 9 36·7	− 1·5	9
7	,, ,, No. 4822	15 40 13·24	+2·21	54 50 21·4	− 0·6	10
9	A.G.Z. Hels. Gotha, No. 8408	15 30 57·10	+2·24	55 37 16·3	+ 0·3	11
,,	,, ,, ,, No. 8461	15 38 15·43	+2·27	55 34 27·4	− 0·1	12
14	,, ,, ,, No. 8240	15 5 10·20	+2·25	57 4 14·4	+ 2·8	13
18	,, ,, ,, No. 8172	14 51 29·71	+2·18	57 33 23·2	+ 4·2	14
18	,, ,, ,, No. 8183	14 54 6·69	+2·18	57 25 48·2	+ 4·2	15
19	,, ,, ,, No. 8149	14 48 10·35	+2·17	57 48 16·8	+ 4·7	16
20	,, ,, ,, No. 8117	14 45 12·33	+2·14	57 44 30·4	+ 5·1	17
24	,, ,, ,, No. 7966	14 22 16·29	+2·04	58 3 56·9	+ 6·2	18
25	,, ,, ,, No. 7949	14 19 35·56	+1·97	58 6 44·2	+ 6·9	19

Comet 1904 *I.*—(*continued*).

Date. 1904	Star's Designation or Authority for Place.	Mean R.A. Equinox of Year. h m s	Corrections to Mean R.A. s	Mean Declination Equinox of Year. ° ′ ″	Correction to Mean Declination. ″	No. of Reference.
May 25	A.G.Z. Hels. Gotha, No. 7988	14 25 45·86	+2·02	58 10 54·4	+ 6·5	20
28	,, ,, ,, No. 7774	13 54 48·05	+1·76	58 11 40·3	+ 7·9	21
June 3	,, ,, ,, No. 7709	13 44 0·44	+1·55	57 44 8·9	+ 9·0	22
4	,, ,, ,, No. 7657	13 36 51·08	+1·50	57 41 31·9	+ 9·2	23
12	,, ,, ,, No. 7525	13 15 53·44	+1·11	56 27 6·2	+10·3	24
21	,, ,, ,, No. 7397	12 55 28·95	+0·78	55 11 42·0	+10·4	25
28	A.G Z. Harvard, No. 4136	12 41 1·05	+0·57	53 37 16·5	+ 9·8	26
29	,, ,, ...	...	+0·54	...	+ 9·9	26
July 7	,, ,, No. 4104	12 31 46·54	+0·31	52 2 39·8	+ 9·1	27
8	,, ,, No. 4097	12 28 59·14	+0·27	51 46 17·2	+ 8·9	28
8	,, ,, No. 4098	12 29 10·59	+0·27	51 46 22·9	+ 8·9	29
9	,, ,, No. 4095	12 28 36·07	+0·25	51 38 16·7	+ 8·8	30
9	,, ,, No. 4092	12 26 9·86	+0·25	51 30 42·2	+ 8·8	31
11	,, ,, No. 4076	12 21 51·20	+0·18	51 13 49·3	+ 8·3	32
16	,, ,, No. 4079	12 22 34·33	+0·11	50 23 37·0	+ 7·9	33
18	,, ,, No. 4075	12 21 44·42	+0·08	50 1 7·8	+ 7·7	34
		Encke's Comet.				
Nov. 26	A.G.Z. Leipzig I., No. 8555	21 28 25·02	+2·06	+12 10 56·4	+25·8	35
Dec. 5	,, ,, II., No. 10528	20 57 6·34	+1·77	7 9 11·5	+22·6	36
5	,, ,, II., No. 10523	20 56 36·28	+1·77	7 0 42·7	+22·6	37

Date.	Star's Designation or Authority for Place.	Mean R.A. Equinox of Year. h m s	Correction to Mean R.A. s	Mean Declination Equinox of Year. ° ′ ″	Corrections to Mean Declination. ″	No. of Reference.
1904 Dec. 6	A.G.Z. Leipzig II., No. 10508	20 55 14·87	+1·75	6 25 54·2	+22·3	38
8	Paris No. 28996 & Leip. II. No. 10391 ...	20 45 12·63	+1·70	5 11 14·6	+21·5	39
8	A.G.Z. Leipzig II., No. 10387	20 45 3·08	+1·70	5 6 36·3	+21·4	40
10	A.G.Z. Albany, No. 7242	20 38 52·92	+1·65	3 59 33·2	+20·6	41
13	Paris No. 28383 & Albany No. 7168 ...	20 27 46·75	+1·60	2 2 6·2	+19·4	42
14	A.G.Z. Albany, No. 7129	20 23 46·54	+1·58	1 12 20·2	+18·3	43
17	θ Aquilæ (Nautical Almanac)	20 6 21·12	+1·54	− 1 6 23·4	+17·5	44
17	A.G.Z. Nicolajew, No. 5097	20 8 58·40	+1·54	− 0 57 37·7	+17·5	45
	Comet 1905 *II.*					
1905. Jan. 4	Schjellerup, No. 442	1 21 32·24	−0·13	− 5 9 50·7	− 8·6	46
8	A.G.Z. Nicolajew, No. 308	1 31 1·08	−0·09	− 1 53 29·4	− 8·1	47
9	,, ,, No. 300	1 29 6·70	−0·12	− 1 5 14·3	− 7·8	48
9	Bonn Beob. vi. − 1° 207	1 28 38·22	−0·10	− 1 15 42·3	− 7·8	49
11	A.G.Z. Nicolajew, No. 303	1 29 54·63	−0·11	+ 0 28 1·8	− 7·4	50
26	A.G.Z. Leipzig I., No. 612	1 58 0·23	−0·06	+11 53 23·0	− 5·4	51
	Comet 1905 *III.*					
Mar. 30	A.G.Z. Berlin A., No. 1883	5 59 14·15	+0·13	+15 49 18·2	−10·7	52
Apr. 3	,, ,, B., No. 2304	6 13 37·38	+0·12	+20 37 4·5	− 9·0	53
8	A.G.Z. Cambridge, No. 3449	6 36 57·76	+0·14	+26 30 39·2	− 7·0	54

Comet 1905 *V.*

Date. 1905	Star's Designation or Authority for Place.	Mean R.A. Equinox of Year. h m s	Corrections to Mean R.A. s	Mean Declination Equinox of Year. ° ′ ″	Correction to Mean Declination. ″	No. of Reference.
Nov. 20	A.G.Z. Hels. Gotha, No. 14580	23 54 55·25	+4·38	+62 27 46·5	+31·1	55
23	Bonn vol. vi., No. 4882	23 39 2·66	+3·13	+37 23 16·9	+29·2	56
24	A.G.Z. Leiden, No. 10043	23 35 1·41	+2·84	+30 35 17·3	+27·2	57
29	,, ,, No. 11721	23 34 57·39	+2·56	+ 9 32 37·9	+21·1	58
30	A.G.Z. Leipzig II., No. 11713	23 32 33·15	+2·50	+ 7 5 52·1	+20·2	59

Comet 1905 *IV.*

1906.						
Mar. 6	A.G.Z. Albany, No. 4335...	11 37 15·64	+1·11	+ 1 46 11·3	− 8·0	60
8	,, ,, ...	...	+1·14	...	− 8·2	60
8	,, ,, No. 4300...	11 28 12·87	+1·17	+ 1 48 19·3	− 8·1	61
15	,, ,, No. 4299...	11 27 47·75	+1·18	+ 1 51 37·1	− 8·6	62
18	,, ,, No. 4308...	11 29 55·90	+1·19	+ 2 0 57·7	− 8·7	63
20	,, ,, ...	...	+1·20	...	− 8·7	63
Apr. 18	,, ,, No. 4271...	11 20 59·44	+1·10	+ 2 23 23·5	− 8·6	64
20	,, ,, No. 4265...	11 19 55·46	+1·07	+ 2 36 45·9	− 8·5	65
28	,, ,, No. 4264...	11 19 36·16	+1·00	+ 2 26 4·9	− 8·2	66

Auroræ observed in the Parish of Delting, Shetland, from September 1905 *to September* 1906. By the Rev. Alex. C. Henderson, B.D.

1905.

Sept. 3. No unusual features noted.
„ 22. Visible from 8.40 p.m. till 9.25 p.m.
„ 25. Beginning about 7.50 p.m.
„ 26. No unusual features noted.
Oct. 7. A *curtain* aurora, 8.50 p.m.
„ 28. Auroral light shining through clouds.
„ 29. Auroral light again, through clouds.
*Nov. 15. Crimson, with green streamers.
„ 23. No unusual features noted.
Dec. 29. „ „
„ 30. „ „

1906.

Feb. 15. A broad, white arch.
„ 16. No unusual features noted.
„ 22. Extending over half of the sky.
„ 24. Greenish-white.
„ 25. Greenish-white.
„ 26. No unusual features noted.
Mar. 13. „ „
„ 17. „ „
„ 24. At 9 p.m.
„ 26. A bright, but ill-defined arch.
Apr. 10. A broad arch, with streamers.
„ 28. A beautiful display, chiefly between 10 and 11 p.m.

Here follows a gap, caused by the absence of night in this latitude.

Aug. 14. Aurora, and simultaneous shower of Perseid meteors, from 10 p.m. till 1 a.m. on 15th.
„ 15. Diffused aurora behind cloud.
Sept. 15. Between 10 and 11 p.m.
„ 16. 9 p.m., the streamers shooting up from horizon through Ursa Major, and afterwards forming an arch.
„ 17. *Reported* to me as being crimson and yellow. I did not observe *this* aurora.
„ 22. Began in the NNE at 8.30 p.m. (first *yellow*, afterwards light *green*). At 9.30 p.m., flickering clouds extended across the zenith, and there were crackling noises.
„ 23. Greenish-white.

The Manse of Delting, from which these auroræ were observed, is in lat. 60° 24′ ¾ N ; long. 1° 19′ ½ W ; longitude in time, 5 min. 18 sec.

* At 7.40 p m. the crimson aurora extended in a broad band across Altair, and to the south of it. At 9.42 p.m. the magnificent green streamers appeared.